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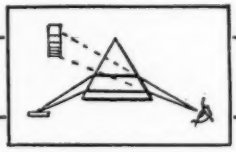

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Journal of the
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A Monochromator for the Near Ultraviolet

FRANK BENFORD, *Research Laboratory, General Electric Company*
(Received January 15, 1936)

THE spectral region from 2000 to 4000 Angstroms has in recent years become of great interest to a rather wide range of scientists, and perhaps no one is more interested than the biologist who has found new and important biological reactions to certain radiations in this region. The incidence of certain wave-lengths on the human skin will produce erythema, or, when in excess, sunburn. The abiotic effect on some low organisms holds promise of a sterilizer for liquids and foods, and there is the extremely important antirachitic effect that still awaits precise evaluation. These and other considerations led to the design and construction some five years ago of a monochromator intended particularly for the "near ultraviolet," and advantage was taken of the then relatively new product, fused quartz, to construct an instrument much more powerful than any instrument then available.

One of the primary considerations in a monochromator for a physical laboratory is chromatic purity. In order to secure this it is customary to make the instrument of the double type, of which the Forsythe[1] instrument is perhaps the most noteworthy example. This double construction is expensive both in money and in energy because of the increased number of optical surfaces. In the present instrument the aim was a maximum of monochromatic energy, with spectral purity and wave-length accuracy of secondary importance.

Fig. 1 gives a general view of the instrument in service in biological research. The anaesthetized rat at the extreme right is in position to receive monochromatic radiation from an arc lamp back of the screens attached to the left arm of the instrument.

OPTICAL DESIGN

Fused quartz in blocks large enough for the Pellin-Broca prisms of this instrument cannot be obtained free from bubbles and striae. From a preliminary inspection of the fused quartz available when this monochromator was designed, it seemed that the matter of stray radiation would be serious in a "single" monochromator design, and that difficulty would be encountered when dealing with the region beyond 3000A, where the energy in the spectrum lines is often relatively weak. It is the general rule, with available sources of ultraviolet and visible radiation, that the average energy per unit of wave-length increases with the wave-length. It is, therefore, to be expected that in any given region of the ultraviolet the stray radiation will come largely from longer wave-lengths than the one being used; and if any portion of the spectrum is to be eliminated as a preventive of stray radiation, the longer wave-lengths should be chosen. Fortu-

[1] Forsythe, Rev. Sci. Inst. **4**, 289 (1933).

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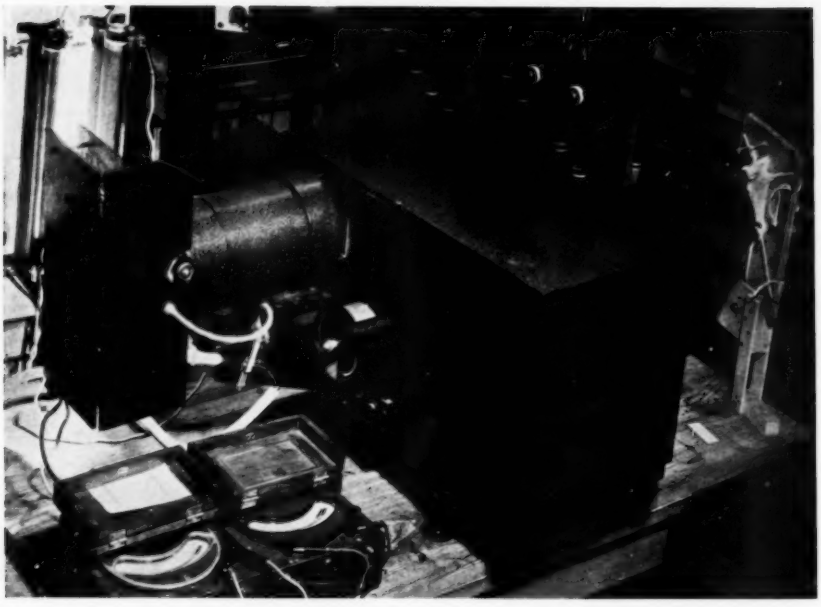

FIG. 1. The monochromator in service with a quartz mercury arc attached at the entrance slit and a rachitic rat receiving monochromatic ultraviolet at the exit slit.

nately, the Pellin-Broca prism lends itself readily to such a scheme, for by changing the angles at the four corners, the unwanted wave-lengths may be largely refracted out of the back of the prism, that is, at the surface that reflects the transmitted radiation, without disturbing the dispersion, which remains that of a normal 60° prism.

The angles are so selected that if the instrument is set to emit the 4000A wave-length at the exit slit, the ultraviolet region is transmitted in full strength; while, the visible region is weakened by the two prisms to about one-fifth of its normal value. The effect of stray radiation from the visible region will, therefore, be reduced to approximately the same proportional value. When set at 4000A, the back-face leakage begins at 4000A, but as the prisms are rotated to bring shorter wave-lengths to the exit slit, the leakage point follows down the spectrum at a slower rate. The curves of Fig. 2 show the weakening of the unused part of the spectrum for two settings of the instrument.

When the instrument is set at 3000A leakage begins at 3400A, and about nine-tenths of the full strength of the visible spectrum is discarded.

The arrangement of the prisms is shown in Fig. 3 where an axial ray is traced through the instrument. At the point in the first prism where internal reflection takes place, some of the incident energy is incident at less than the critical angle and part of this energy is refracted out of the optical system, and therefore eliminated as a potential source of stray radiation. In the second prism the process is repeated, and this emitted radiation is received and absorbed by black shields, not shown in the sketch. The two prisms are arranged so as to be the dispersive equivalent of two sixty-degree prisms in train.

If the instrument is set to emit visible radiation, the axial image that falls on the exit slit is weakened by losses at the two reflecting faces as indicated by the computed curve of Fig. 4. Between the limits 4000A and 7000A the transmitted energy is reduced to about half of full value, but on account of the relatively large amounts of monochromatic energy available, this loss has never been found to detract from the usefulness of the instrument in working with visible radiation.

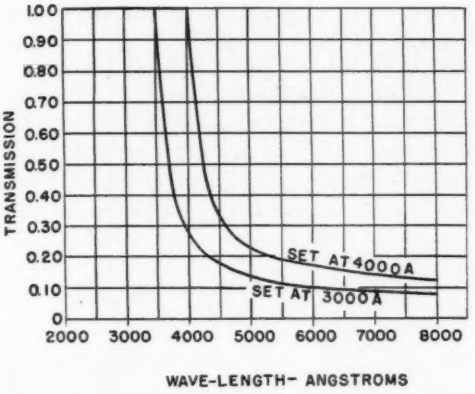

FIG. 2. The transmission of the instrument, as influenced by the internal reflections of the prisms is shown for two wave-length settings, 3000A and 4000A.

Some of the prism dimensions are given below for the smaller of the two prisms (which is the upper one in Fig. 3).

Angle $a = 72° 51' 50''$	Face a–b, 11.55 cm
$b = 94° 16' 16''$	b–c, 17.37 cm
$c = 60° 0' 0''$	c–d, 9.66 cm
$d = 132° 51' 54''$	d–e, 13.71 cm

The angular dimensions of the larger of the two prisms are the same as given above, and the linear dimensions are all about 15 percent greater.

The small prism weighs 7 lbs. 3 oz., and the large one 10 lbs. 10 oz.

Each collimating lens is composed of two simple lenses, as illustrated in Fig. 3. These lenses have spherical faces that were designed for $n = 1.48690$ at 3000A. The curvatures of the four faces were computed to give four equal refractions to a marginal ray, and the spacing of faces was computed to satisfy the time equation between the marginal ray and the axial ray. The collimators are always used wide open (f:2 at 3000A) throughout the range 2500A–8000A, and the only diaphragms used are for the purpose of suppressing internal reflections in the collimating tubes.

Two shutters are provided to cut off the spectrum when desired. Both are internal, and are between the collimating lenses and the prisms. This location was selected so that a piece of plate glass can be used to cut off radiation. The glass shutter transmits most of the scattered radiations of wave-lengths greater than 3300A, and this permits the elimination of this energy from the galvanometer net deflection, which is the difference between the deflections for the shutter open and closed. Both metal and glass shutters are used, and as the stray radiation is weak in comparison with the stronger mercury lines, the metal shutter is used for a greater part of the time.

The first slit is curved to attain a straight line image at the exit slit, and the energy can thus be imaged on the straight line of junctions of the thermopile.

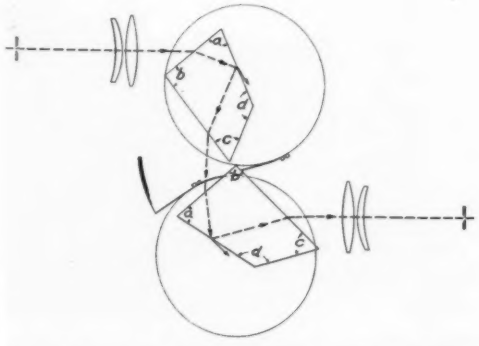

FIG. 3. Fused quartz monochromator. The broken line shows the optical path of the axial ray, and the eliminated energy is indicated by the short arrows at the rear of each prism.

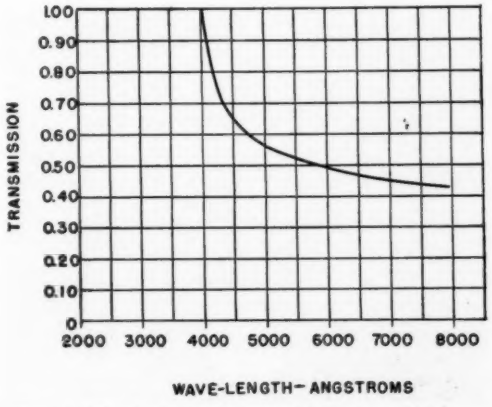

FIG. 4. The instrument transmission for axial images considering only the internal reflection at the rear of each prism, is 100 percent up to 4000A, and averages 50 percent in the visible spectrum.

Mechanical Design

A single aluminum alloy casting forms the support for all the optical parts and the entrance and exit slits. In the design care was taken to make the extensions of the casting that carries the collimating tubes heavy enough to support an arc lamp or any other auxiliary device that it might be desired to attach to the monochromator. In many physical measurements it is desirable to have a thermopile that can be removed from the exit slit and then returned to exactly the same position. This can best be accomplished by using a swinging arm to attach the thermopile to the collimator head, as is shown in Fig. 5. The same can be said about the quartz arc, which must be brought into a horizontal position to start, Fig. 6, and must be returned to a fixed operating position as in Fig. 7.

The slits are bilateral and are fixed in position with a maximum opening of 15 mm width and a height of 40 mm. The collimating lenses are mounted on a wide base that is screw driven, with the handle of the screw set well back out of the way under the heads.

The posts that carry the prism turntables are carefully turned one-inch diameter steel bars, held in deep bosses in the bedplate, and clamped in such a way that tightening the clamp will not tilt the post. A one-inch hardened steel ball rests upon the head of each post, and the turntable rests on this ball as a bearing. The hub of the turntable comes down about three inches over the post and the assembly gives a sturdy and free-moving support for the prisms.

The driving mechanism for turning the prisms consists of a precision feed screw carrying a drum with a linear scale, and a push rod from the end of the feed screw to a bearing near the rim of one turntable. A steel belt passes around a short arc of each turntable and then to a heavy spring attached to the mount of the feed screw. The belt is clamped to each turntable to prevent

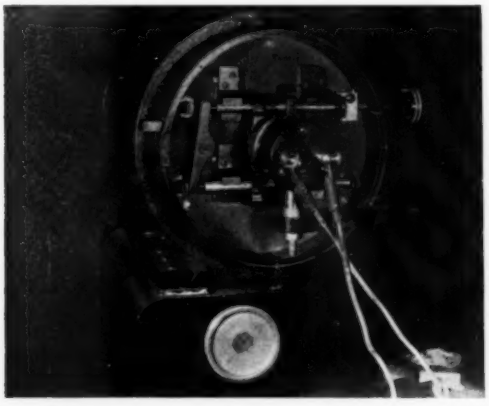

Fig. 5. This photograph shows a line thermopile held against the exit slit by a swinging arm attached to the monochromator.

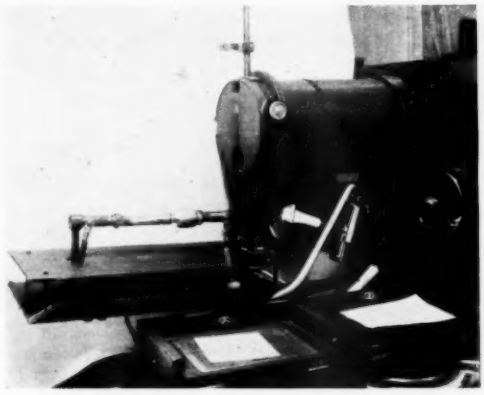

Fig. 6. The quartz mercury arc that is ordinarily used is shown in the starting position, with its points of support attached to the collimator casting.

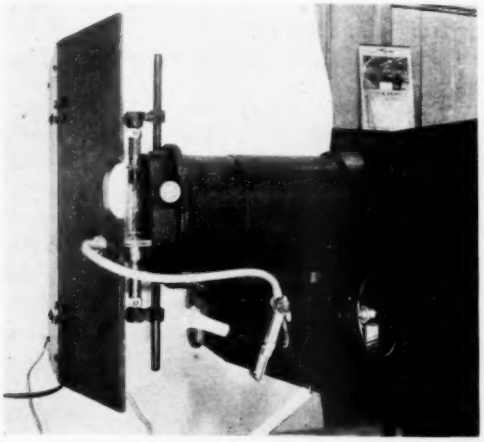

Fig. 7. The arc lamp is shown in operating position within a quarter inch of the entrance slit. The rubber hose and glass nozzle are used to control the lamp voltage by means of a stream of cooling air directed against the neck of the lamp.

FIG. 8. A top view of the instrument with the cover opened to show the two prisms mounted on their turntables. Interior shields have been removed to give a better view of the prisms. The thermopile shown at the exit slit swings away to permit the rat being brought into line with the slit.

slipping, and the spring effectively prevents backlash.

As the prisms are rather massive and heavy, each is accurately counterbalanced by a brass weight of the same plan section, symmetrically placed on the opposite side of the turntable. Fig. 8 gives a view of the two prisms, and shows the general optical arrangement.

OPTICAL CHARACTERISTICS

The drum on the feed screw that drives the turntable has a scale 150 centimeters long, divided into millimeters, and reference is had to a calibration curve for any desired wave-length. The relation between drum indication D (centimeters) and wave-length λ (Angstroms) can be represented by the equation

$$(\lambda-\lambda_0)(D-D_0)=P \tag{1}$$

and similarly, the dispersion curve of fused quartz can be represented by

$$(\lambda-\lambda_0)(n-n_0)=k, \tag{2}$$

where n is the refractive index as found in tables of indices.

Both of these expressions are much simpler than is customary for a dispersion equation because only a short range of wave-lengths (from 2650 to 4000A), is considered at one time, and a similar simple expression is used for the range 4000 to 7000A.

From angular measurements on the rotation of the prisms there is evidently a linear relation between D and the angle of rotation, θ, and we have

$$aD=\theta. \tag{3}$$

Let f_3 be the designed focal length of the collimators at 3000A, and let n_3 be the refractive index at the same wave-length. At some other wave-length the focal length f is

$$f=f_3(n_3-1)/(n-1). \tag{4}$$

The rate of change of D with respect to wave-length λ is, from (1)

$$dD/d\lambda=-P(\lambda-\lambda_0)^2. \tag{5}$$

Also from (3) we get

$$adD=d\theta \tag{6}$$

and the dependent variable in (5) can be

changed to

$$d\theta = -aPd\lambda/(\lambda-\lambda_0)^2. \quad (7)$$

The separation of lines measured at the slit and in a plane perpendicular to the axis of the second collimator is

$$S = 4\int f d\theta, \quad (8)$$

where the factor 4 is introduced because the ray of minimum deviation rotates 4 times as fast as the prisms.

From (2) we have

$$n-1 = k/(\lambda-\lambda_0)+n_0-1 \quad (9)$$

and substituting in (4)

$$f = f_3(n_3-1)/(k/(\lambda-\lambda_0)+n_0-1). \quad (10)$$

Substituting (7) and (10) in (8) we get

$$S = -4af_3P(n_3-1)\int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{[k/(\lambda-\lambda_0)+(n_0-1)](\lambda-\lambda_0)^2}$$

$$= \frac{4af_3P(n_3-1)}{k} \log \frac{(\lambda_2-\lambda_0)[k+(n_0-1)(\lambda_1-\lambda_0)]}{(\lambda_1-\lambda_0)[k+(n_0-1)(\lambda_2-\lambda_0)]}. \quad (11)$$

The constants have the following numerical values.

$\lambda_0 = 1547.2$	from (2)	$k = 59.041$	from (2)
$n_0 = 1.44623$	from (2)	$a = 0.000361$	from (3)
$D_0 = 155.52$	from (1)	$n_3 = 1.48690$	from tables of indices
$P = 123{,}870$	from (1)	$f_3 = 29.7$	from design.

The values substituted in (11) gives 2.2844 cm as the separation of lines 2652 and 3650 in the mercury spectrum. This wide separation makes it possible to use one-mm slits in working with the eight stronger mercury lines in this region.

The focal surface is of course a curved surface making a large angle with the plane of the exit slits, and only the central ray will focus in the plane of the slit for any given setting. The edge rays focus either inside or outside of the slit and sharp spectral boundaries are impossible. The lack of focus makes it inadvisable to open the exit slit much beyond 100 Angstroms. There is undoubtedly some merging of energy from adjacent lines, but in biological work, where the errors in individual measurements are usually large, this spectral impurity is not important.

When working with a continuous spectrum, or with a strong line with adjacent weak lines, this method of computing the wave-length emitted from a wide slit is of much practical value.

Typical Data

Some typical figures are tabulated below for a quartz "Uviarc" at 3.3 amperes and 150 volts.

Wave-length	2652	2804	2894	2967	3021	3125	3650
Microwatts	100	300	200	400	700	1400	3400

It should be noted that 2652 is weaker in relation to 3650 than is normal with this arc. This is due to the absorption of the shorter wave-lengths by the fused quartz which totally extinguishes the resonance line 2536.

It will be recognized by those familiar with monochromatic energies that the above values represent an entirely different order from the usual spectral power that may be obtained by means of a monochromator. Experiments that previously were difficult if not impossible have been done readily with this instrument and several of the design features have been found by experience greatly to facilitate research work.

The Validity of the Photographic Reciprocity Law for Alpha-Rays

R. N. WOLFE AND T. R. WILKINS, *Department of Physics, University of Rochester*

(Received September 18, 1935)

The validity of the photographic reciprocity law for alpha-rays has been confirmed for a ratio of source intensities of 171 to 1. The law had previously been confirmed for a ratio of source intensities of only 7 to 1. The source intensities used were equivalent to 1.38 millicuries/cm² and 8.06×10^{-3} millicurie/cm². The reciprocity law is found to hold for this ratio of source intensities when the exposures are made in a sequence which eliminates the effect of latent image growth. To avoid complications due to aggregate recoil the sources were coated with a layer of silver approximately 0.1μ in thickness.

INTRODUCTION

AN apparent failure of the reciprocity law for alpha-rays at normal temperature was found in a study of the variation of the alpha-ray response of a photographic emulsion with temperature made by Sheppard, Wilkins, Wightman and Wolfe.[1] For this study an alpha-ray sensitometer was needed. It was desirable that this instrument have two alpha-ray sources with an intensity ratio of about 80 to 1; the weak one to give the lower part of an H. and D. curve and the strong one the upper part, because the total exposure time must be as short as possible. An instrument containing two such sources was made up and the intensity ratio was determined photographically, i.e., the ratio of the exposure times necessary to produce equal densities was determined. It was intended to measure the absolute intensities with an ionization chamber and an amplifier at a later time. It was thought valid to do this because Kinoshita[2] and Salbach[3] had reported that the reciprocity law held for alpha-rays and theoretically[4, 5] it should be expected to hold.

When the absolute determinations were made it was found that the ratio of the intensities determined electrically differed from the ratio of intensities determined photographically.

Electrical Ratio 67/1, Photographic Ratio 80/1.

This variation was outside the limits of experimental error and, therefore, it must be explained. This variation may be explained in one of several ways:

1. Either the reciprocity law did not hold for alpha-ray exposures.
2. Or the alpha-ray sources transferred activity to the film itself.
3. Or some other radiation was also emitted for which the reciprocity law failed.
4. Or growth or decay of latent image gave a spurious effect.

Since it had become a fundamental assumption in photographic theory that the reciprocity law of photographic exposure held for radiation having a large amount of energy per quantum such as x-rays and also for alpha-rays, any possibility of the failure of this law for alpha-rays must be carefully checked. This was done directly by using electrically calibrated sources of radically differing strength.

HISTORICAL

The application of the reciprocity law to alpha-rays

S. Kinoshita[2] checked the reciprocity law over a 6.5 to 1 range and H. Salbach[3] checked it over a 7.3 to 1 range. W. Bothe[6] and J. Eggert and F. Luft[7] claimed that it was valid but did not specify over what range it was valid. These ranges are exceedingly small compared to the range of the intensities studied in the case of light (10^6 to 1 in some cases). G. Briggs[4] in 1927 stated that since $D=D_{\max}(1-e^{-cN})$ over a range of N of 180 to 1, where N was varied by varying the exposure time, the reciprocity law

[1] Sheppard, Wilkins, Wightman and Wolfe, to be published.
[2] S. Kinoshita, Proc. Roy. Soc. **A83**, 432–453 (1909).
[3] H. Salbach, Zeits. f. Physik **11**, 107–128 (1922).
[4] G. Briggs, Proc. Roy. Soc. **A114**, 313–340 (1927).
[5] Wilkins and Wolfe, J. O. S. A. **23**, 324 (1933).
[6] W. Bothe, Zeits. f. Physik **13**, 106–112 (1923).
[7] Eggert and Luft, Zeits. f. physik. Chemie (Bodenstein-Festband) Erganzungsband, 745–754 (1931).

was valid for a ratio of source intensities of 180 to 1. He did not make a direct check of the reciprocity law because he used a source of only a single intensity to check this equation. Wolfe[8] found by varying the intensity of a source of alpha-rays with a rotating sector that the reciprocity law held over a range of 10 to 1. This might be interpreted as checking not the reciprocity law but the effect of intermittent exposure. However, when one considers the small probability that two alpha-rays will strike a single grain during one sector opening, it is seen that a failure of the reciprocity law cannot be distinguished from the effect of intermittent exposure. Since this is true the rotating sector data are not conclusive. Thus it can be seen that the only direct experimental evidence as to the validity of the reciprocity law is found in the work of Kinoshita[2] and Salbach[3] cited above.

The photographic reciprocity law failure with exposure to light

Since it will be of interest later to compare the magnitude of the alpha-ray effects with those produced in the study of the reciprocity law failure for light, the following data for light given by V. Parks[9] are quoted. The densities given are for equal energies falling on the emulsion at different levels of intensity.

The densities cited in Table I are of the same magnitude as those obtained in the present experiments and the intensity ratios are even smaller than those used in these experiments. Contrast bromide emulsion is of the same type as process emulsion. It does, however, show a slightly higher reciprocity law failure.

TABLE I. *Contrast bromide emulsion.*

λ	Density High Intensity	Density Low Intensity	$D_{High}-D_{Low}$
	Ratio of $\frac{\text{High Intensity}}{\text{Low Intensity}}=\frac{123}{1}$		
4500A	2.35	2.02	0.33
4500A	2.85	2.54	.31
4000A	1.32	1.08	.24
	Ratio of $\frac{\text{High Intensity}}{\text{Low Intensity}}=\frac{40}{1}$		
4500A	2.20	1.81	.39
4000A	1.26	1.00	.26

Experimental

Alpha-ray source preparation

In order to make a direct check of the validity of the reciprocity law for alpha-ray exposures two sources of radically different strengths must be prepared. The sources must emit only homogeneous alpha-rays and no other radiation. One of the sources must also be very active in order to give a useful density in a few minutes. Since polonium emits only homogenous alpha-rays two polonium sources are suitable. One of the sources must be as strong as possible and the other one must be as weak as can be used without necessitating excessively long exposure times. The strong source used had a polonium density of 1.38 millicuries/cm² and the weak source had a polonium density of 8.06×10^{-3} millicurie/cm². This gave an intensity ratio of 171/1. The polonium was secured from old radon needles kindly furnished by Dr. C. O. Hansen of the Strong Memorial Hospital, Rochester, New York, and Dr. M. C. Reinhard of the State Institute for the Study of Malignant Disease, Buffalo, New York. It was deposited from a reducing solution of H_2SO_3 following the method of Whitaker, Bjorksted and Mitchell.[10, 11]

It is interesting to note at this point that the strongest polonium source of which the authors found a record was one produced by Erbacher[12] in 1933. It had a polonium density of 39.1 millicuries/cm². Since it would be difficult to prepare stronger sources than this and since the use of weaker sources involves excessive exposure times (days or weeks) it does not seem possible to study the reciprocity law for alpha-ray exposure over the range that the law has been studied for light.

Silver covering over sources

Since there was uncertainty as to the tenacity with which the polonium stuck to the silver and

[8] R. Wolfe, Master's Thesis, University of Rochester, 1932.
[9] V. Parks, Master's Thesis, University of Rochester, 1932.
[10] Whitaker, Bjorksted and Mitchell, Phys. Rev. **46**, 629 (1934).
[11] Private Communication from M. D. Whitaker to T. R. Wilkins, October 22, 1934.
[12] O. Erbacher, Zeits. f. physik. Chemie (A) **165**, 421–426 (1933).

also since the sources used in the previous check of the reciprocity law were covered with an evaporated layer of silver it was thought desirable to coat these sources with silver also. Dr. T. A. Russell kindly evaporated a layer of silver on them using the method developed by Dr. Russell and Dr. Brian O'Brien of this laboratory. A piece of glass alongside of the sources when the evaporation was made was coated until it had a diffuse optical density of of 0.50. This corresponds to a film approximately 0.1μ in thickness. This is not necessarily the silver film thickness over the sources, but is probably a lower limit to the thickness.

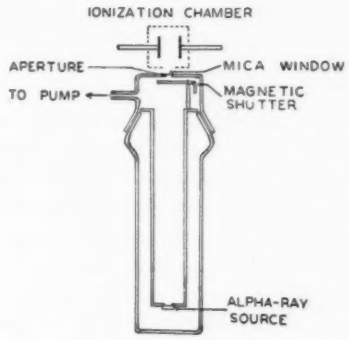

FIG. 1. Apparatus used for calibrating alpha-ray sources.

Source calibration

In order to check the validity of the reciprocity law of photographic exposure for any radiation it is essential that the relative intensities of the two sources be accurately known. Since the absolute intensities of the sources can be readily determined in terms of the number of alpha-rays emitted per minute the relative intensity of the sources was determined by measuring their absolute intensities and taking the ratio of these intensities.

The method of measuring the absolute intensity of each source was as follows: A source was placed in a vacuum chamber containing an aperture, a magnetic shutter, and a mica window as shown in Fig. 1. The mica window had a stopping power of approximately 1.4 cm air equivalent. The air pressure in the tube was sufficiently low so that the alpha-rays on emerging from the mica window could pass into the ionization chamber. This ionization produced a current which was amplified with an amplifier of the Wynn-Williams type[13] until it operated a mechanical counter. The amplifier circuit is shown in Fig. 2. Counts were made with the sources at varying distance, and also with different apertures and counting rates. The strong source emitted 3080×10^6 alpha-rays/cm^2/min. and the weak source emitted 18×10^6 alpha-rays/cm^2/min. These figures were used in calcu-

[13] Wynn-Williams and Ward, Proc. Roy. Soc. **A131**, 391 (1931).

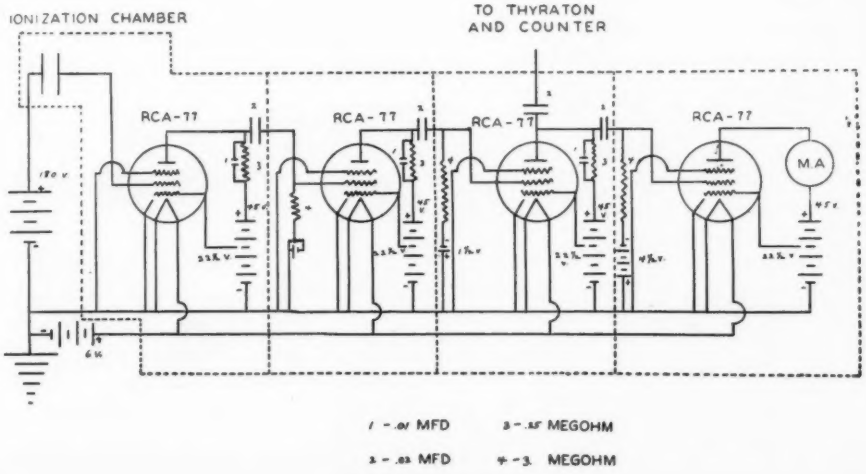

FIG. 2. Amplifier circuit.

lating the quantity of polonium on the sources which is given above.

Apparatus

A diagram of the apparatus used for making the exposures is shown in Fig. 3. *W* and *S* are the two calibrated polonium sources. They are set so that they are each equidistant from the photographic film *F*. This distance from source to film is 4.25 ± 0.01 cm. It is desirable that the distance be the same in order to eliminate any inverse square law correction and simply give exposures in the inverse ratio of the source intensities. Two magnetically controlled shutters are indicated at S_1 and S_2. The photographic film shown in Fig. 4 was held against a plate by clamp *C* so that it lay accurately equidistant from and parallel to the sources. The whole apparatus was placed inside a bell jar. The two magnetic shutters were controlled by power sent in through two wires waxed into an opening in the bell jar. A return lead was provided by a screw which also controlled clamp *C*. Successive exposures were made by releasing clamp *C* and moving the strip of film with another screw. The film was placed in position under clamp *C* the shutters S_1 and S_2 closed. Then the bell jar was pressed onto the base using stopcock grease as the sealing medium. Then the pressure was lowered to 2 mm of mercury or less depending on the quality of the seals which the bell jar made against the base plate and the vacuum screws made in the bell jar. As previous work[5] has shown luminosity effects are negligible at 2 mm pressure and since only 0.1μ has been removed from the range of the alpha-rays in the emulsion at this pressure, a pressure of 2 mm Hg would be satisfactory. The pump was always run until an equilibrium pressure was reached before exposures were made.

The photographic film used in this experiment was Process film on nitrate base, emulsion number 6–206, 8015. Film was chosen instead of plates because film is coated more uniformly with emulsion than are plates. All films were developed in D–19 (x-ray) developer for 4 minutes at between 19°C and 20°C. The developer was placed in a wide mouthed bottle which would accommodate the films used. The films were developed by placing them in this bottle, stoppering the bottle and tilting it continuously by hand. This was done slowly enough so that little frothing of the developer was produced. It was hoped this would reduce oxidation effects.

Diffuse densities were used in this experiment. They were measured with a Capstaff-Purdy densitometer.

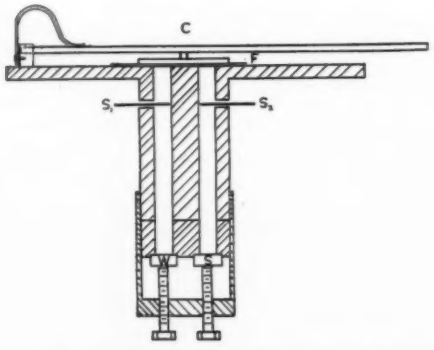

FIG. 3. Apparatus used for studying the photographic reciprocity law for alpha-rays.

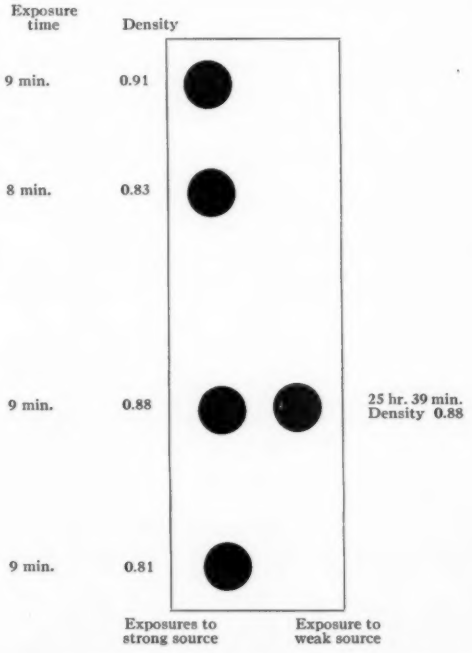

FIG. 4. A typical exposed and developed film. (No. 18.) Exposures were made consecutively from the top of the film. (The 8 minute exposure is not shown on Fig. 5.)

Results

As is stated above, the reciprocity law is valid if a given number of alpha-rays falling on a photographic plate always gives the same density regardless of the rate at which the alpha-rays strike the plate. It was previously found that the strong source gave a density of approximately 0.80 in 8 to 9 minutes. A series of exposures was run on a single film in which the strong source was exposed for 8 minutes and the weak source was exposed for 171×8 minutes or 22 hours 48 minutes. The factor 171 is the ratio of source intensities determined by the electrical counting method. Later 9 minutes was used for the strong source exposure and 25 hours 39 minutes for the weak source exposure.

Fig. 5 shows in diagrammatic form the results obtained when exposures were made in various sequences. The width of the heavy lines correspond to the duration of the strong source exposures and the horizontal lengths of the dotted lines correspond to the duration of the weak source exposures. Table II gives the complete experimental data which are shown graphically in Fig. 5.

Film No. 15 Fig. 5 shows the first experiment made with the calibrated sources. The reciprocity law seemed to check very well as indicated by the equal densities. Upon more careful examination of the diagram of Film No. 15, however, it is observed that the two exposures did not receive identical after-treatment because they were developed immediately after the strong source exposure was made. In order to check any possible change Film No. 17 was made. A strong source exposure was made at both the beginning and the end of the weak source exposure. The strong source gave a higher density when exposed 23 hours before development than when exposed immediately before development. The weak source density was intermediate between the strong source densities. This indicated that the reciprocity law held but that a growth of the latent photographic image might easily cause spurious effects which might be mistaken for failure of the reciprocity law. Films No. 18, No. 19, and No. 23 taken under more extreme conditions confirm this view. On each of these films one of the strong source exposures was made when the weak source exposure was one-half completed. This should partially compensate for latent image growth. The following densities were produced for these exposures.

TABLE II.

	Weak Source Exposure		Strong Source Exposure	
	*Time between Exposure and Development**	*Density*	*Average Time between Exposure and Development*	*Density*
Film No. 15	A. 22 hr. 50 min.	0.86	10 min.	0.86
	B. 2 min.			
Film No. 17	A. 22 hr. 56 min.	0.85	22 hr. 52 min.	0.88
	B. 8 min.		4 min.	0.80
Film No. 18	A. 46 hr. 19 min.	0.88	52 hr. 55 min.	0.91
	B. 20 hr. 40 min.		33 hr. 33 min.	0.88
			5 min.	0.81
Film No. 19	A. 35 hr. 57 min.	0.83	47 hr. 54 min.	0.88
	B. 10 hr. 18 min.		47 hr. 43 min.	0.88
			23 hr. 8 min.	0.86
			6 min.	0.79
Film No. 23	A. 36 hr.	0.78	43 hr. 11 min.	0.80
	B. 10 hr. 21 min.		43 hr.	0.79
			23 hr. 10 min.	0.76
			17 min.	0.72
			7 min.	0.72

* A. Time between beginning of exposure and development.
B. Time between end of exposure and development.

	Weak Source Density	Strong Source Density	$D_s - D_w$
Film No. 18	0.88	0.88	0.00
Film No. 19	.83	.86	+0.03
Film No. 23	.78	.76	−0.02

These differences are within experimental error and are much less than those produced for even smaller ratios in the case of light. Compare these results with those of V. Parks[9] for light which are cited above. Thus consistent results which check the validity of the reciprocity law are obtained when exposures are made in this manner.

Latent image growth

When the exposures are not made in this manner, however, a growth of latent image appears which might easily be mistaken for a failure of the reciprocity law. It is a growth of latent image because the order of exposure determines whether the strong source or the weak source will produce the greater density as may be seen in Fig. 5. Now an apparent latent image growth could have been produced by the deposition of some radioactive material on the film at the

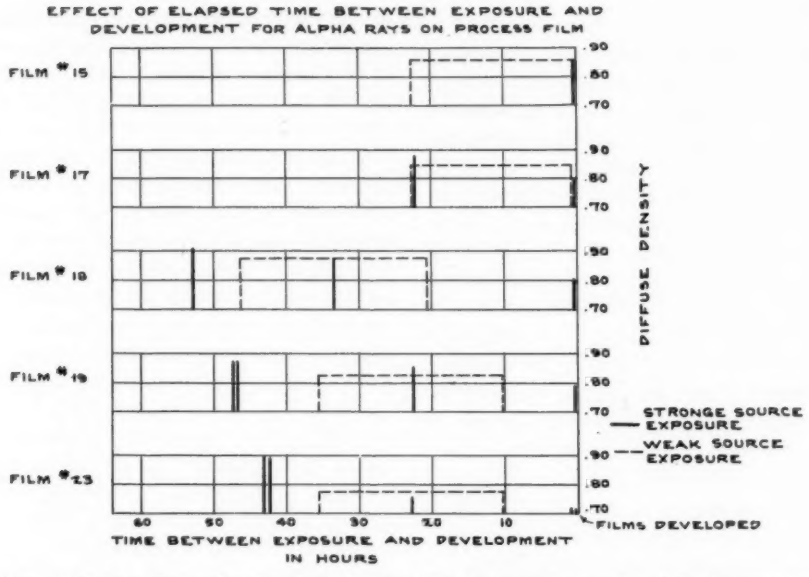

FIG. 5. Diagram illustrating the effect of elapsed time between exposures and development for alpha-ray exposures on Process film.

same time that it was bombarded by the alpha-rays. The radioactive material would continue to emit radiation thus exposing the film after the exposure was thought to have been stopped. Lawson[14] has reported such radioactive recoil and also evidence of it was found in some preliminary work of this experiment. In this earlier work some radioactive contamination in the apparatus was traced to recoil of polonium from the sources. However, at the time that Films No. 18, No. 19, and No. 23 were made, a strong source exposure was made for 48 hours. This exposure was sufficient to produce a print-out image. Even with this exposure there was not sufficient radioactive material deposited on the film by recoil to cause the observed growth of latent image. Since only 9-minute exposures to the strong source were used for this experiment there seems to be no reason to believe that the growth was caused by radioactive recoil. Therefore the latent image growth is similar to the growth of latent image found in the case of exposure to light. Such a growth for alpha-rays was reported by M. Blau.[15]

SUMMARY

The reciprocity law of photographic exposure is found to hold for a ratio of alpha-ray source intensities of 171–1 when the exposures are made in a sequence which eliminates the effect of latent image growth.

The authors wish to express their sincere appreciation to the various members of the staff of the Eastman Kodak Company Research Laboratory for their many helpful suggestions and criticisms.

[14] R. Lawson, Akad. Wien. Sitz. (IIA) **128**, 795–830 (1919).

[15] M. Blau, Akad. Wiss. Wien (IIA) **140**, 623–628 (1931).

The Reflection and Absorption of Daylight at the Surface of the Ocean

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AND

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IN previous measurements of the penetration of daylight into the sea considerable doubt has existed as to the magnitude and importance of the light "lost" at the surface of the water through reflection or absorption under the various conditions of sea and sky.[1–10] Attempts to measure the light present just beneath the surface of the water for comparison with that just above the surface have usually been unsuccessful because the action of the waves and the motion of the ship have made it impossible to keep the submarine photometer in the desired position long enough to make a measurement. Furthermore, the light for several meters below the surface fluctuates so rapidly that it is very difficult to obtain a reliable reading at these levels particularly when using the potentiometer method. The result is that the depth of the first point of observation has often not been less than 5 meters in off shore localities and occasionally no satisfactory measurement could be obtained short of 10 m. When the curves representing the penetration of light from the first point downwards are extended upwards to zero depth, it is commonly found that they strike the abscissa at points indicating a reduction in the light at the very surface of from a few percent to 50 or 60 percent. This discrepancy has been termed the "surface loss."[10]

It was the purpose of the present investigation to determine first of all whether such large surface losses actually exist or whether they can be accounted for by some instrumental or observational error. Secondly it was desired to distinguish the loss due to reflection at the surface from the apparent loss which would be produced by a stratum of less transparent water which might have existed undetected just beneath the surface. Finally it was hoped that some definite relations could be worked out between the amount of surface loss and the degree of roughness of the sea surface, the clearness of the sky, and the altitude of the sun. If this could be done it would be possible to use the transmissive exponent once obtained for a given locality at a given season for calculating the actual irradiation present at any desired depth under any conditions of weather or sea surface from the value of daylight intensity at the surface—either measured on the spot or assumed from standard meteorological observations.

The incident light I at the surface of the sea may be divided into three components: A, that which penetrates the surface and is entirely absorbed in the sea; R, that which is reflected either at the smooth surface of the water or by white foam and bubbles on the surface; and U, that which penetrates the surface but is scattered back by particles and bubbles in the water below and finally returns to the air. Therefore,

$$I = A + R + U.$$

If the sea is covered with foam, the surface must be defined as the surface of the bubbles of the foam.

The magnitudes of the three components A, R, and U may be determined by placing a photometer in various positions relative to the sea surface and comparing the values obtained with I, the intensity found with the receiving surface (a diffusing disk of opal glass) of the photometer placed upwards and horizontal. If the instrument

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* Contribution No. 98.

[1] Zenker, *Die Verteilung der Warme auf der Erdoberflache* (Berlin, 1888).

[2] Paul Regnard, *La vie dans les eaux* (Paris, 1891), pp. 204–217.

[3] V. E. Shelford and F. W. Gail, Publ. Puget Sound Biol. Sta. **3**, 160 (1922).

[4] Shelford, Publ. Puget Sound Biol. Sta. **7**, 161 (1929).

[5] H. H. Poole and W. R. G. Atkins, J. Marine Biol. Ass. **15**, 482 (1928).

[6] Poole and Atkins, J. Marine Biol. Ass. **16**, 313 (1929).

[7] Birge and Juday, Trans. Wisconsin Acad. **24**, 574 (1929).

[8] W. R. G. Atkins and H. H. Poole, J. Marine Biol. Ass. **17**, 617–632 (1931).

[9] Atkins, J. Conseil International pour l'Exploration de la Mer **7**, 191 (1932).

[10] Atkins and Poole, Phil. Trans. Roy. Soc. London **B222**, 144–148 (1933).

is suspended over the sea in an inverted position, it measures the quantity $R+U$ which is the total light coming upward from the sea's surface. If the photometer is lowered in an inverted position to a point just beneath the surface, U can be measured alone.

The total light passing downwards through the surface is $A+U$, but this quantity cannot be measured directly with precision as already explained. However, since I, U, and $R+U$ can all be accurately measured, the values of A and R alone can be calculated. The quantity $R+U$ is of particular interest as being that fraction of the light falling on the ocean which does not contribute to its supply of energy. The magnitude of this fraction has not been determined previously to our knowledge.* The remaining light, which is the quantity A, is the important component in relation to the ocean water itself. A is absorbed by molecules of water, by bubbles, by suspended particles, and by organisms. This component is the entire source of heat for the ocean (if infrared radiation is included) and is responsible for the photochemical reactions of marine organisms—especially photosynthesis, without which life in the sea would not be possible.

Floating Photometric Tower

The light passing upwards from the surface of the sea fluctuates very rapidly with the motion of the waves. It was therefore necessary to use all possible means to integrate this flicker over time. The first step was to mount the photometers high above the water and far from any obstruction which would cut off the light from the sea. Accordingly a light framework of pipe was constructed in the shape of a triangular pyramid, made buoyant by lashing a barrel to each corner. The sides of the base measured 4.6 m, and the height of the legs was 6.7 m. Framework and barrels were painted black to minimize reflection into the two photometers which were secured in an inverted position at the top of the tower at a vertical distance of about 6 m above the surface of the water. A 23 kg concrete block attached to each corner of the base lowered the center of gravity of the tower so that it could stand a 30° tip without capsizing. Since the tower floated with its triangular pipe base about 20 cm under water, it had very little effect upon the surface of the sea even in the strong winds of Buzzards Bay. The tower with all its equipment weighed about 90 kg and was easily lifted out of the water onto the stern of the power boat, *Asterias*, from which the measurements were made. In this way it was carried to location and then lifted overboard. Two rubber covered cables 100 meters long ran between the two photometers and the recording apparatus on the boat. The tower floated sufficiently far away so that the boat cut off a negligible amount of reflected light.

The performance of the tower was entirely satisfactory. Waves approximately 1.2 m high and a wind of nearly 20 miles per hour tossed it around so little that it was not necessary to mount the photometers in gimbals. Towards the end of the summer some of the pipes had become considerably weakened due to the action of sea water at the joints, and the tower collapsed while being lifted aboard. A tower of similar dimensions but made of angle iron welded at the joints would be much stronger and have the same advantages as the pipe construction.

The photometers employed were those used previously by Oster and Clarke[11] except that the photoelectric cells in the sea photometers were exchanged with those in the deck photometers. The heavy sea photometers were mounted on gimbals and placed facing upwards on deck while the deck photometers were mounted on the tower. The combinations of cells and filters employed covered the "red" region (6000–7000A), and the "violet" region (3100–4500A). The tower photometers were calibrated against those used on deck before each set of observations as previously described.[12]

The Electrical Circuit

Since the light from the surface of the water flickers rapidly the "marine potentiometer"[13] which gives an instantaneous value of its intensity could not be used for these measurements.

* Except for a few measurements made from an aeroplane (see below).

[11] Oster and Clarke, J. Opt. Soc. Am. **25**, 85 (1935).
[12] G. L. Clarke, Biol. Bull. **65**, 317 (1933).
[13] Gall and Atkins, J. Marine Biol. Assn. **17**, 1017–1028 (1931).

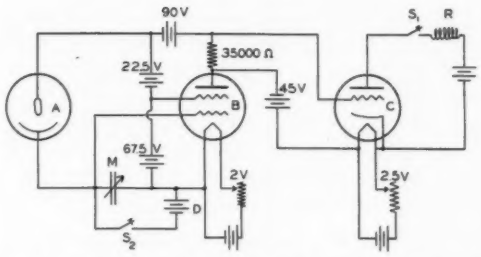

FIG. 1. Circuit employing a Thyratron tube used for integrating variable photoelectric current.

The suitability of the neon discharge-tube photometer[13] was tested by connecting it in series with the potentiometer in order to measure the current by both methods simultaneously as described by J. H. J. Poole.[14] While the potentiometer gave a constant reading to within one percent with the cell under constant illumination, the number of counts from the neon tube varied by at least 10 percent under all conditions of capacity and voltage. The response of the neon tube was found to be nonlinear with change in current which made it impossible to use in a flickering light.

A circuit employing a Thyratron (Fig. 1) was found to be entirely satisfactory. The photoelectric cell A is connected in series with a 90-volt battery and a variable condenser M with 0.5, 0.05, 0.005, and 0.0005 microfarad capacity. The battery side of the condenser is connected to the filament of a 232 amplifier tube B. The other side goes to the control grid of the tube, and also to relay switch S_2. The battery D makes the control grid negative when the relay switch is closed. A resistance of 35,000 ohms in the plate circuit of the 232 tube gives a change in voltage between the filament and control grid of the 885 Thyratron tube C. This tube is filled with a low pressure of argon. When the control grid is negative no current flows in the plate circuit. As the control grid becomes more and more positive it reaches a point where a gaseous discharge takes place between the filament and the plate. The plate current rises to a very high value, and is no longer affected by the grid potential. The plate circuit contains a relay R and relay switch S_1.

The circuit operates as follows. The current from the photoelectric cell charges up the condenser M until the voltage on the control grid of the 232 tube produces a large plate current. This current increases the voltage across the 35,000-ohm resistance until the grid of the 885 tube starts the discharge. The relay is excited and performs three operations. First it opens switch S_1 thereby extinguishing the discharge in the 885 tube. Second it closes switch S_2 bringing the control grid of the 232 tube back to the voltage determined by the battery D. Third it moves the escapement of a watch so that the second hand records half a second. The whole process repeats itself, and the watch keeps a count of the number of discharges.

To calibrate the circuit in amperes it is necessary to know its capacity with the photoelectric cell connected and the difference between the voltage delivered by D and the discharge voltage. The former is measured by plotting the time for a fixed number of discharges produced by a steady current against various capacities of condenser M. The intersection of the curve with the capacity axis gives the circuit capacity. The discharge voltage can be found by replacing D by a potentiometer.

The precision of the circuit is limited by two factors, first by the range of uncertainty in the tripping voltage, and second by the control grid current of the 232 tube. The latter can be reduced to a minimum by choosing a tube with a low grid current, and the former can be diminished to an unimportant fraction of the whole by increasing the range through which the voltage of the condenser runs. The 885 tube could be used alone except for the fact that the uncertainty in the tripping voltage is about 1.5 volts. Care must be taken that the counts be not too fast as the relay takes about 0.01 second to react and therefore introduces a one percent inaccuracy in counts one second apart. The range of the circuit can be extended by increasing the voltage of C or the capacity M. The leakage current is about 2×10^{-11} ampere and is quite steady so that a current of 10^{-10} ampere can be measured to one percent. The largest current is about 10^{-5} ampere.

The tubes and batteries are mounted in a strong wooden box which is almost air tight and is kept dry inside with activated alumina. The

[14] J. H. J. Poole, Sci. Proc. Roy. Dublin Soc. **19**, 17–25 (1928).

box is opened occasionally to change the capacity M. The 885 tube uses a storage battery which plugs into the box and the relay, in a separate box of its own, plugs into the circuit box. The wire from switch S_2 to condenser M enters the relay box through a hard rubber petticoat insulator. In foggy weather it is often necessary to pour very hot paraffin over the insulator and the ends of the leads from the photoelectric cells to stop surface leakage.

Two wires lead to each deck cell, but in the case of the tower cells only one lead goes to each cell from the boat. The other lead is grounded to the tower, and the electrical circuit is completed through the water to a metal electrode hung over the side of the boat. The resistance of the sea water for a distance of 100 meters cannot be more than 200,000 ohms and therefore has no effect on the readings. Care must be taken to standardize with all the cable in the water since the capacity of the circuit changes on submersion.

New Photometers Employing Rectifier Cells

Following the general lines of other investigators[15–17] two photometers were constructed by using the Westinghouse Photox Cell (Type SW–30) with a microammeter made by the same company (Type PX–4) and a suitable shunt (Fig. 2). The full-scale reading of the microammeter is 25 microamperes, and the internal resistance for this particular instrument is 297.2 ohms. No. 1 Photox cell was mounted in sea photometer case No. 2 with its face 4 cm from the inside surface of the heavy plate-glass window which was covered on the outside with an opal glass diffusing disk as before. For use on deck a small cylindrical copper box was constructed at the bottom of which cell No. 2 was secured at a distance of 6 cm from the receiving window which consisted of a similar diffusing disk.

The needle of the microammeter is so very highly damped that even in the stormiest weather the effect of the motion of the boat is negligible.

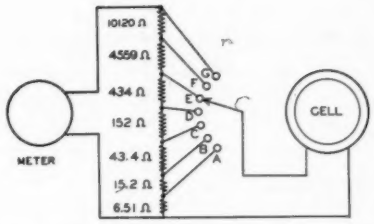

Fig. 2. The circuit for the photometers employing the Photox cell and microammeter showing the connections for the variable shunt.

It is necessary to have at least 3000 ohms in the external circuit to avoid over damping the needle. This has the disadvantage of demanding high resistances in the shunt and the external circuit of the Photox cell except when the shunt is set at G. Consequently the current-intensity curve deviates very markedly from a straight line, and at high intensities runs almost parallel to the intensity axis. To avoid using this part of the curve a diaphragm was placed on top of the diffusing disk of each photometer whenever the intensity demanded it. As a result only shunts E, F, and G need be used.

The spectral sensitive of our Photox cells has not been specially determined, but since the manufacturer reports that these cells do not vary greatly, it seems safe to use the published average curve.[18] The average sensitivity curve shows a maximum at 5500A, dropping off to values of ten percent of the maximum at 4650A and 6250A, and therefore corresponds closely with the "green" region investigated by Oster and Clarke.[11]

Photox cell No. 2 was calibrated for each setting of the shunt by comparing its response with that of the vacuum photoelectric cell[19] used

[15] Atkins and Poole, J. Marine Biol. Ass. **19**, 67–72 (1933).

[16] Poole and Atkins, Sci. Proc. Roy. Dub. Soc. **20**, 537 (1933).

[17] Petterson and Landberg, Göteborgs Kungl. Vetenskaps-och Vitterkets—Samhalles Handl. **B3**, No. 7 (1934).

[18] Westinghouse Instruments Catalog, Section 43–705, 1935.

[19] As an added precaution all four vacuum cells were tested for their linearity of response to intensity in the following manner, H. Kreusler, Ann. d. Physik **6**, 412 (1901)). A lens focused the light from a 500-watt lamp on the photometer diffusing disk. Two doors were placed between the photometer and the lens close to the lens. When both doors are open the brightness of the image on the diffusing disk is I. With door 1 closed the brightness is I_2 and with door 2 closed it is I_1. With both closed the light coming through the crack between them gives a brightness I_b. The optical arrangement makes $I=I_1+I_2-I_b$. If the cell is linear then the corresponding currents will add. The method has the advantage that it is not necessary to know what the various intensities are but only whether the corresponding currents add. All four photoelectric cells were found to be linear to within 1 percent.

on the tower, but with the green filter (4900–6200A) used previously. The cells to be compared were placed so that they received the same amount of light from two 500-watt bulbs. This was checked by exchanging their positions. The currents from both cells were measured simultaneously, and then the cells were moved further away from the lights so that the intensity changed by the same amount for both cells, and the currents measured again. Three separate graphs were made of the Photox cell current against the photoelectric cell current corresponding to the three settings of the shunt, *E*, *F*, and *G*. By means of these curves referring the nonlinear response of the Photox cell to the linear one of the vacuum cell, the determination of all relative intensities with the Photox cell is made possible. The calibration was accurate to within 1.5 percent.

Photox cell No. 2 was then placed beside No. 1 and diaphragms chosen which made both give the same current. When moved together away from or towards the light, the two currents were always equal to each other. This showed that the emission of cell No. 1 followed the same curve as that for cell No. 2. A further test was made to ascertain whether the position or size of the spot of light falling on the diffusing disk had any effect upon the response of the cells. A lens was set up in front of a 500-watt lamp so as to bring the image of the filament to a focus in space. The cells were moved about in the beam of light in such a way that the luminous flux striking the diffusing disk always remained the same, but the size and position of the spot of light varied over all possible cases. No change in current was observed so long as the total luminous flux remained the same.

Since rectifier cells are known to be affected by temperature changes, the emission of the sea photometer was tested at 26.7°C, 20.5°C, and 12.0°C under three illuminations covering the range of intensities met with in actual use. The greatest deviation found was an increase in emission at 12.0°C over that at 26.7°C of 13 percent under conditions of high illumination. Changes in temperature of this magnitude are not ordinarily encountered during any one series of measurements at sea. However, in those cases in which the temperature of the air is significantly different from that of the water, this source of error is avoided by allowing the sea photometer to cool down to the temperature of the water before beginning the measurements.

The Photox deck photometer was standardized against a 1000-watt incandescent lamp of known emission in the same manner as previously described.[12] The limits of spectral sensitivity of this photometer were arbitrarily taken as 4650A and 6250A. The irradiation within this range at a distance of 1 meter from the lamp is 310μw/cm^2 (about 1500 meter candles*) and produced a current of 13.5μ amp. with shunt *F*. Through the use of the relative calibration curves already described the intensity of light corresponding to any reading of the instrument can be determined in absolute units. The weakest light that can be measured (1μ amp. with shunt *G*) is 7μw/cm^2 (about 34 m.c.). The sea photometer, which is always compared with the deck photometer before and after every series of measurements, has almost exactly the same emission.

In using the Photox photometer for measuring the reflection from the ocean, the light deck case was attached to the end of a long bamboo pole, and held some 4.5 m out over the side of the boat. The pole was turned so that the window of the photometer was directed first up and then down. Alternate measurements were made of I and $R+U$ as rapidly as possible, and in unsteady conditions the average of six or more alternate readings taken. Tests showed that slight deviation of the photometer from the horizontal, either in the upright or the inverted position, resulted in negligible variations in the response of the instrument provided that the altitude of the sun was not less than 20°. The value of $R+U$ did not change significantly with changes in the vertical distance of the photometer above the water from the highest accessible position of 3.5 m down to 1 m under conditions of smooth or slight sea.

In the use of the submarine photometer the accuracy of such measurements as could be made just under the surface was greatly enhanced by marking the cable supporting the instrument at measured small intervals with white lead. An

* Since the spectral sensitivity of the Photox cell corresponds closely to that of the human eye, the standardization of this photometer in meter candles does not involve as great an error as in other parts of the spectrum, reference 12.

TABLE I. *Values of the total light coming upward from the surface of the water ($R+U$) as a percentage of incident light (I).* Measurements made with the violet and the red sensitive photoelectric photometers on the floating photometric tower.

DATE 1935	WIND—WAVE	SKY	TIME (Local apparent time)	ALTITUDE OF SUN	$R+U$ AS PERCENTAGE OF I Violet	Red
July 3	SW Force 3	Clear	1233	70°	4.2%	2.5%
Buzzards Bay	Height of waves about	Clear	1338	62	4.1	2.4
	0.3 to 0.6 m	Clear	1433	53	4.3	2.5
	Same	Clear	1448	50	4.6	2.6
	Same	Clear	1518	44	4.8	2.9
	Same	Clear	1548	39	5.1	3.6
July 10	Slight ripple	Many clouds, sun out	1123	69	11.2	
Buzzards Bay	No wind	Overcast	1218–1428	70–53	6.4 (Average)	
	Oily, no wind	Many clouds, bright sun	1458	47	5.9	
	Smooth	Full sun	1518	44	5.7	
	Light air	Full sun, many clouds	1600	36	6.8	
	Light breeze	Full sun, few clouds	1633	30	6.5	
	Light breeze	Full sun, few clouds	1708	23	8.4	
	Light breeze	Almost full sun, few clouds	1733	18	7.6	
	Light wind	Overcast	1748	16	6.7	
	Light wind	Sun through clouds	1808	12	6.7	
	Light wind	Overcast	1833–1903	8–0	6.5 (Average)	
July 23	SW Force 3	Hazy sun	1118	68	9.3	5.7
Buzzards Bay	Waves 0.5 m	Slightly hazy sun	1203	68	7.7	3.6
	Waves 0.5 m	Clear	1243	65	6.4	5.6
	Waves 0.5 m	Clear	1318	62	5.8	6.3
	Waves 0.6 m	Clear	1345	56	6.2	7.0
	Waves 0.6 m	Many clouds	1433	49	8.4	9.9
	Waves 0.6 m	Many clouds, sun in and out	1508	42	8.2	10.7
August 8	NW Force 3–4	Clear	1048	61	3.8	3.4
Vineyard Sound	NW Force 3–4	Clear	1123	64	3.9	3.4
August 8	NW Force 3–4	Clear	1508	41	6.4	5.0
Buzzards Bay	NW Force 3–4	Clear	1538	36	7.3	5.3
	NW Force 3–4	Cloud over sun	1543	35		4.3
	NW Force 3–4	Clear	1613	29	9.1	7.0
	NW Force 3–4	Clear	1648	23	12.0	10.3
	NW Force 3–4	Clear	1718	17	14.2	20.5
	NW Force 3–4	Clear	1748	12	15.4	19.0
	NW Force 3–4	Clear	1823	5	12.7	35.2

observer was stationed to watch the cable and readings were taken only at those moments when as a result of the motion of the boat and the water a designated mark on the cable coincided with the surface.

OBSERVATIONS

The measurements of the total upward light, or $R+U$, made with the red and violet sensitive photometers on the floating photometric tower (Table I) show that only a slight difference exists in the values for these two components of daylight. $R+U$ for red light is almost always somewhat lower than for violet, as would be expected from the fact that the index of refraction for water is a little higher for the latter than for the former part of the spectrum. The green component was investigated exclusively with the Photox photometer except for one occasion when measurements were made with the green sensitive photoelectric photometer to compare the two methods. The instruments agreed to well within the experimental error and showed that the value of $R+U$ for green light is intermediate between red and violet.

$R+U$ reaches a minimum (about 4 percent for violet and 3 percent for red) on clear days when the sun is nearest the zenith; on cloudy days the value may reach 6 or 7 percent. After the sun has fallen below about 44° from the horizon the value of $R+U$ begins to rise. Below 30° on a clear day $R+U$ becomes steadily higher, and is higher then than under any other conditions. From 12° on the inaccuracy introduced by the unavoidable tilting of the

photometers invalidates the measurements but there is every indication that $R+U$ becomes very high as predicted by theory. For solar altitudes of 12° to 30° $R+U$ ranged between 6 and 20 percent in clear weather. Whitecaps were found to increase $R+U$ only slightly, apparently because the foam produced covers a relatively small proportion of the total surface. This is in agreement with a value of 12 percent obtained over the foamy part of the boat's wake. Although the magnitude of the upward light has been found to remain an inconsiderable fraction of the incident light under all conditions, it must be remembered that its percentage may be two or more times greater on one occasion than on another and its absolute value varies with the strength of daylight.

Since these results point to a much lower value for the loss of light at the surface than was indicated by previous measurements of submarine illumination, an attempt was next made to measure with more care and precision than before the intensity of the light just below the surface. The Photox photometers were used for this purpose. Because of the high damping of the microammeter needle the deflection for a rapidly varying light gives a much steadier reading than with the potentiometer method, and the results are therefore much more reliable. All submarine measurements have been multiplied by the factor 1.09 to correct for "external internal reflection."[10]

Under conditions of absolute calm, clear sky, and glassy smooth water in Buzzards Bay it was possible to make measurements up to within 10 cm of the surface where 97.5 percent of the light was transmitted (Series 401, Fig. 3). The trend of the curve is straight from this point to a depth of 2.7 meters. When the curve is extrapolated to zero depth a value of about 100 percent is reached. The theoretical reflection (not including U) for the existing solar altitude of 40°, as calculated from Fresnel's law of reflection for a smooth water surface, is about 2.5 percent. The "surface loss" indicated by extrapolating a series of measurements which extend nearly to the surface should be compared with the true reflection R and not with $R+U$ because U is a "loss" only as far as the deeper strata are concerned. Accordingly, the agreement of these

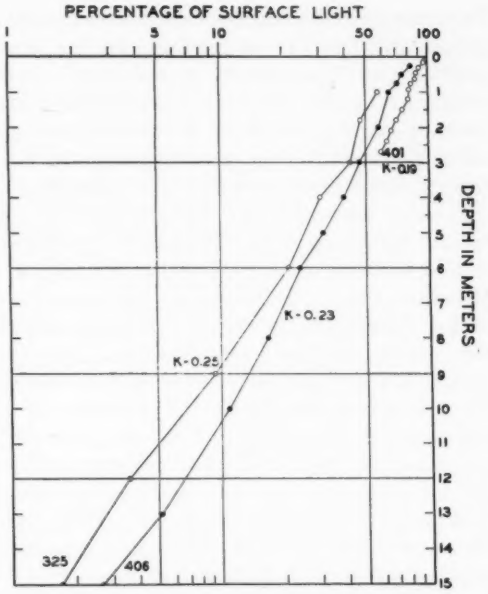

FIG. 3. Relation between depth and irradiation expressed as a percentage of the light just over the surface (logarithmic scale). Series number indicated at end of each curve. Series 325: Buzzards Bay, Aug. 28, 1934. 0926–1011 L.A.T. Av. altitude of sun 47°. Slight sea, few clouds. Green sensitive photoelectric photometer. Series 401: Buzzards Bay, Aug. 15, 1935. 1508–1538 L.A.T. Sun 40°. Glassy smooth sea, clear sky. Green sensitive Photox photometer. Series 406: Provincetown Harbor, Aug. 24, 1935. 1300–1335 L.A.T. Sun 54°. Slight sea, clear sky. Photox photometer.

measurements with theory is good, as is the case also with the succeeding series of observations. A short time later when the sun was about 35° from the horizon and the theoretical reflecting power was about 5 percent, an actual measurement of $R+U$ gave 8.4 percent. In this case therefore U amounted to about 3.4 percent or almost half of the total upward light.

The situation is entirely different when the surface of the sea is disturbed as is revealed by a comparison with Series 325 made at the same place the year before[11] on a day when small waves existed. If the curve is extrapolated beyond the 1 meter level, a "surface loss" of about 30 percent is indicated. Since the value of the transmissive exponent for the upper 6 meters ($k=0.19$) is identical with that for Series 401, the difference in the position of the two curves cannot be due to a difference in the transparency of the water on the two occasions.

Furthermore, we know from our present investigation that $R+U$ could not have been as great as 30 percent. It was therefore suspected that the stratum of water less than 1 meter thick just beneath the surface became relatively opaque when the surface of the sea was rough and that this condition disappeared in calm weather.

Such a situation was actually detected in measurements made in Provincetown Harbor (Series 406) on a clear day with wind force 4 (Beaufort Scale), small waves, and occasional flecks of foam. If the curve is extrapolated to zero depth using only points below 1 meter it indicates a surface loss of 15 percent or more. But when one extrapolates from points above 0.5 meter a loss of only 6 percent is found. In this case $R+U$ was 4.6 percent and U, measured directly by inverting the sea photometer and lowering it a few centimeters beneath the surface, was 3 percent. The abrupt change of slope in the curve reveals the increased rate of absorption and scattering within the first meter of water.

On Georges Bank (Station 2473) the roughness of the sea caused a regular heave of the sea photometer of 3 meters and prevented measurements from being made above 2 meters. The wind was force 3 and there were a few whitecaps. The curves for both the violet component (Series 410) and the green component (Series 408) of daylight extrapolate to 84 percent at the surface (Fig. 4). The value of $R+U$ was 5.5 percent, and of U measured 1 meter down was 2 percent.

South of Cape Cod (Station 2474) in smoother water with waves about 1 meter high, a wind of force 4–5, and numerous whitecaps, it was possible by watching the cable very carefully to make a measurement at 0.5 meter (Series 412, Fig. 4). Without this point the surface loss appears to be 14 percent, while with it the loss is reduced to almost nothing. $R+U$ was 4.6 percent.

The upward component of light beneath the surface was measured with an inverted photometer at each meter to a depth of 14 m in Provincetown Harbor where the depth is about 16 m. At 0.5 m the upward component was found to be 4 percent of the downward component and this ratio increased gradually to a value of 6

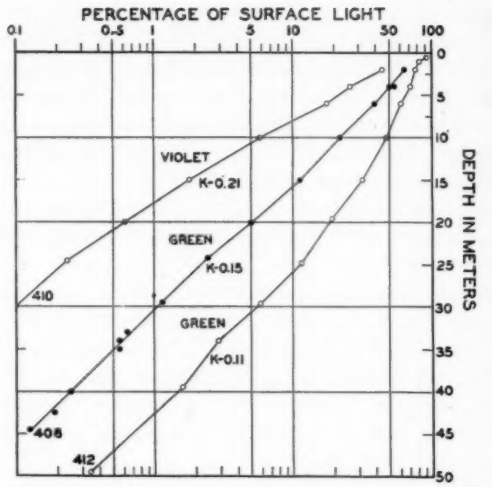

FIG. 4. Observations made on Georges Bank (Station 2473: Lat. 41° 51′, Long. 68° 44′). Aug. 25, 1935. Series 408: 1150–1250 L.A.T. Sun 59°. Moderate sea, many clouds. Photox photometer. Series 410: 1350–1450 L.A.T. Sun 47°. Moderate sea, overcast. Violet sensitive photoelectric photometer. Observations made south of Cape Cod (Station 2474: Lat. 40° 48′, Long. 71° 00′). Aug. 26, 1935. Series 412: 1214–1259 L.A.T. Sun 59°. Moderate sea, clear sky. Photox photometer.

percent at 14 meters. In Buzzards Bay the ratio was found by Dr. Oster the year before to be 4.8 percent at 1 m, 5.8 percent at 5 m, and 8.9 percent at 10 m. In the deep basin of the Gulf of Main (Station 2472: Lat. 41° 53′, Long. 69° 25′) we found a ratio of 2.5 percent at 0.5 m and this value remained unchanged to a depth of 20 m.

A few approximate measurements of the horizontal component of the submarine illumination were made by suspending the sea photometer by one end so that the receiving surface was vertical.[8] The photometer was caused to rotate slowly around a vertical axis in order that the window might face successively in all directions relative to the direction of the sun which was unobscured. A maximum and minimum intensity was thus recorded for each depth. In Provincetown Harbor the ratio of the maximum value for horizontal illumination at 1.5 m to the vertical downward illumination at the same depth was 16 percent and the ratio of the minimum horizontal illumination to the vertical illumination was 3.4 percent. At 10 m these ratios had increased to 17 percent and 7 percent, respectively. In the

deep basin of the Gulf of Maine the ratios were 6 percent and 3 percent at 2 m, and 20 and 3 percent at 15 m. From these rough measurements and those of the vertical upward light, it is seen that the illumination becomes more and more diffuse with increasing depth but the directional character of the light is lost only slowly.

Conclusion

From the foregoing observations we may conclude that the discrepancies which have been termed "surface losses" do in fact exist but that they are not due chiefly to a "loss" at the very surface. They are due, however, to a stratum of relatively opaque water which exists just beneath the surface and which has generally escaped notice because of the difficulty of making precise measurements at shallow depths. The only portions of the incident light which are "lost" are the component which is reflected from the surface R and the component which escapes upward from the water U and our measurements have shown that the magnitudes of these components are relatively small.

The average value of all our measurements of $R+U$ for solar altitudes greater than 30° is about 6½ percent for cloudy days and about 4 percent for clear days. Differences in the condition of the sea surface ranging from flat calm to waves with whitecaps has very little effect on these figures. The value of $R+U$ was not more than a few units higher in the more opaque inshore waters than in the clearer water of the Gulf of Maine. $R+U$ is never more than 9 percent except when the sun's altitude is less than 30° with a clear sky. These conditions would exist during only a small portion of the day for most of the ocean, but the greater reflection losses at lower solar altitudes is undoubtedly of considerable importance in polar seas. For values of $R+U$ of 9 percent or less, R and U were found to be about equal in magnitude. Therefore, the portion of daylight which never enters the water at all, namely R, is never more than a few percent of the incident light and consequently of negligible importance.

These results form an interesting comparison with the measurements of Kimball and Hand[20] of the reflectivity of different kinds of surfaces made from an aeroplane in the vicinity of Washington, D. C. near noon on three days in January, February, and May. These investigators flew over bare grourd, fields, forests, rivers, and Chesapeake Bay mostly at altitudes greater than 1000 ft., and recorded simultaneously the light from an overcast sky and from the terrain below. In no case was the reflection found to be greater than 11 percent except where considerable snow existed. A maximum value of 89 percent was recorded over a very white snow field. Green fields and bare ground reflected 6 to 10 percent, dark forests about 4 percent, and light forests about 9 percent of the incident light. Values of 5 to 10 percent were obtained over the Potomac River (at altitude of 10–20 ft.) and of about 6 percent over the Patuxent River. The reflection from Chesapeake Bay on the January trip when the surface was smooth was 9.7 percent (altitude 2000 ft.), but in May, when whitecaps existed, the values obtained varied from 3.4 to 4.9 percent (altitude 2000–3000 ft.). No significant difference was found between the measurements near shore and those made more than 1½ miles out where conditions approximating the ocean were believed to exist. It is extremely satisfactory that these observations agree so closely with ours which were made only a few feet above the surface. They also agree with the reflection from a grass lawn measured in the quadrangle of the Biological Laboratories, Cambridge, at 1.30 P.M. E.S.T., Nov. 4, 1935, where we obtained 10 percent reflection in the sun and 9 percent in the shade with the Photox deck photometer.

Under conditions of absolute calm the transparency of the water remains unchanged up to within at least 10 cm of the surface. The "surface loss" deduced from extrapolation is very small and in agreement with the low magnitude of the reflection. But as the sea's surface becomes more and more disturbed, the "surface loss" discrepancy becomes larger and larger. Its magnitude is usually about 15 percent[5, 8] but has often been reported as much greater. Since we have found that the value of $R+U$ does not become significantly larger under rough conditions than in calm weather, it follows that the amount of light absorbed in the upper meter or so has

[20] H. H. Kimball and I. F. Hand, Monthly Weather Review, U. S. Weather Bureau **58**, 280 (1930).

greatly increased. Careful investigation of this stratum has proved that this is indeed the case. The abrupt changes in slope near the surface in our transparency curves which prove this fact are also to be seen in the curves of other investigators in those few cases where measurements have been made at depths of less than one meter.[10]

The cause of this diminished transparency of the surface layer is undoubtedly the presence of bubbles produced by a disturbance of the water. Chemical substances, particles of sediment, and planktonic organisms are uniformly distributed over a much greater depth than the thickness of this anomalous stratum by the vertical mixing produced by turbulence. In every case where a large "surface loss" has been recorded either white caps or a tide rip has existed. In calm weather the phenomenon disappears. From any high point on a boat the presence of clouds of bubbles below the surface may be observed which are produced by whitecaps and remain under water long after the disturbance has passed. These bubbles reduce the light by scattering and multiple reflection.

Summary

(1) Measurements were undertaken to ascertain the cause of the large "surface losses" which have been found in the investigation of submarine illumination and to determine the influence of different conditions of sky and sea on the magnitude of such losses.

(2) The light "lost" from the ocean may be expressed by $R+U$ in the equation $I=A+R+U$ (where I=incident light, A=light absorbed by the water, R=light reflected from the surface, and U=light which enters the water but is reflected out again). The value of $R+U$ was investigated by securing photoelectric photometers in an inverted position at the top of a floating tower. The photoelectric current was measured with an integrating device employing a Thyratron tube and relay.

(3) A new sea photometer and a new deck photometer were constructed by using the Westinghouse Photox Cell (rectifier type) and a highly damped microammeter. These instruments were carefully tested and standardized and found extremely satisfactory for the measurement not only of reflection but also of the illumination just beneath the surface.

(4) The value of $R+U$ in Buzzards Bay reaches a minimum of about 3 to 4 percent on clear days when the sun is nearest the zenith. The percentage is increased by one or two units when waves exist and by a few units more when the sky is cloudy, and is decreased by a few units in clearer off shore waters. $R+U$ is never more than 9 percent for solar altitudes greater than 30° in clear weather. R and U are about equal in magnitude.

(5) The greater part of the "surface loss" was found to be accounted for by a stratum of relatively opaque water extending to a depth of about 1 meter and caused by bubbles in this stratum the abundance of which is proportional to the roughness of the sea surface.

On the Refractive Index Changes in Optical Glass Occasioned by Chilling and Tempering

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(Received February 8, 1936)

IN a recent paper Hopfield and Watkins[1] have published observations on the variation of refractive index through heat-strengthened plate glass, but they do not give any figures to indicate the magnitude of the effect which they have observed. The effect was, however, noticed and examined in considerable detail a number of years ago by the present writers.[2] The glasses then examined were optical glasses, and for one type in particular, *viz.*, Dense Barium Crown, a lowering of refractive index by as much as 0.004 was found.

Not only was the lowering of refractive index by chilling observed, but detailed observations were made of its recovery by tempering at various temperatures. Thus, while keeping at the annealing temperature[3] for two hours restored the full value, more than 90 hours at a temperature 70°C lower failed to do so.

The following quotation from the paper cited[2] summarizes the observations made:

1. It has been established that the effect of chilling the D.B.C. and B.S.C. samples referred to may be the lowering of the refractive index by as much as 0.004 in the case of the former and 0.0013 in the case of the latter.

2. This lowering of refractive index is removable by heating to a temperature and for a length of time which have been ascertained for certain cases.

3. It has been found in the case of the D.B.C. that if the glass be cooled from the annealing temperature slowly under pressure and measured not under pressure, the resulting refractive index is higher than when it is similarly cooled free from pressure, by amount of the order of 0.00035.

[1] J. J. Hopfield and G. B. Watkins, J. O. S. A. **26**, 24 (1936).

[2] F. Twyman and F. Simeon, J. Soc. Glass Technol. **7**, 199 (1923).

[3] F. Twyman, J. Soc. Glass Technol. **1**, 6 (1917), i.e., the temperature at which the internal stresses existing in the glass are reduced to 1/20 of their original amount in 0.26 minute. This temperature is distinctly below the softening point of the glass.

The Optical Constants of Several Metals in Vacuum*

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(Received December 16, 1935)

RECENT advances in the technique of evaporation have made it easier to prepare reflecting surfaces which are superior to those obtainable by polishing the bulk metal. The evaporation method is especially applicable to the elements which are so rare that it is impossible to obtain samples large enough to make a usable mirror and a few others which are difficult to sputter. Since the measurement of the optical properties of many substances is made uncertain and even impossible by their tendency to tarnish, the ideal procedure is to prepare and study the surfaces in a high vacuum chamber. The polarization methods used by Drude[1] and Voigt[2] are rather bulky to be included in a vacuum chamber and the possibility of double refraction due to strain in windows makes it undesirable to place any of the essential optical parts outside the chamber. The arrangement used by the Duncans[3] for the alkali metals involves the use of a glass to metal surface which presents the dangers of strains and trapped gas at the boundary where the light is reflected.

With these objections in mind a method was devised, closely resembling that of Brewster[4] who over a century ago observed that polarized light which had been reflected a number of times from a pair of metal mirrors at certain angles of incidence could be extinguished by a Nicol prism. His observations were not correctly interpreted as they preceded by a number of years the full development of the theory which was first given by Kettler.[5] Brewster's experimental method was simpler than those used later as it did not require the use of a Babinet compensator to measure the phase change between the perpendicular and parallel components introduced by reflection from a metal surface. The phase change between the components of the incident polarized light introduced by p reflections from a metal surface at the same angle of incidence is p times the phase change due to one reflection and if the reflected light can be extinguished by the analyzer then the total phase change is also an integral multiple of π. The phase change due to one reflection is this quantity divided by the factor p. The original arrangement of Brewster is shown in Fig. 1(A) and the present modification in (B). A reflecting mirror m used at normal incidence so that it introduced no phase change between the components was substituted for the second Nicol and one Nicol serves as both analyzer and polarizer. This also gives twice as many reflections and makes it possible to dispense with one of the two metal mirrors. If the phase change due to two reflections from M is 180° then the phase change on a single reflection is 90° which occurs only for an incident angle known as the angle of principal incidence and usually designated by ϕ. The mirror m was rotated on an arm about the same axis as the metal mirror M and through twice the angle by means of a two-to-one gear ratio. By using a piece of thin plate glass coated on both sides with the element to be studied, the angles of principal incidence for reflections on each side of the normals to both surfaces could be observed. These angles were in pairs 180° apart and differing by 2ϕ. There were also four orientations of the Nicol which gave extinction for each angle of principal incidence. These were also in pairs 180° apart and separated by 2ψ. The angle ψ, called the principal azimuth, is the angle between plane of incidence and the electric vector of the polarized incident light for which the reflected light is circularly polarized. Extinction was observed by means of a half-silvered mirror H placed outside the vacuum chamber.

The optical assembly was fastened to a steel plate S, twelve inches square and one inch thick, mounted in a vertical plane. Holes were drilled through this plate for the vacuum pump connection, optical windows, mechanical leads to rotate

* Presented at the Philadelphia meeting of the Optical Society in October 1935.

[1] Drude, Wied. Ann. **39**, 481 (1890).

[2] Voigt, Ann. d. Physik **14**, 936 (1904).

[3] R. W. Duncan and R. C. Duncan, Phys. Rev. **1**, 306 (1913).

[4] Brewster, Phil. Trans. **69**, 133 (1830); Pogg. Ann. **21**, 219 (1830).

[5] Kettler, Wied. Ann. **3**, 83 (1878); **22**, 204 (1884).

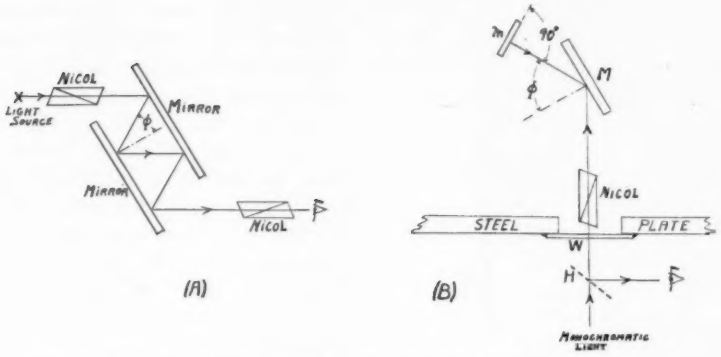

FIG. 1. Methods of measuring optical constants. (A) Brewster's method. (B) Vacuum modification.

the Nicol prism and the mirror system, as well as nine electrical leads. The filament and high voltage leads were of three-sixteenth-inch brass rod insulated from the steel plate by standard Isolantite bushings and sealed with lead gaskets and Glyptal. These made strong and vacuum tight joints and carried large currents without trouble. Mirrors were arranged for illuminating and reading the divided circles through windows in the steel plate. The vacuum chamber was completed by a glass bell jar pressed against the steel plate and the joint covered with Apiezon *Q* compound. With a two-stage mercury diffusion pump and liquid air trap pressures of 5×10^{-6} mm were reached. The light source was a high pressure capillary quartz mercury arc immersed in a water bath. This arc which was recommended by Dr. L. J. Heidt of the Chemistry Department proved far more intense than commercial arcs and was easily constructed. The strongest lines of the arc; 4046, 4358, 4916, 5461, and 5780 (5790 plus 5769) were separated by means of a monochromator.

Experience showed that the greatest variations in the measurements on most elements were due to lack of uniformity among a number of surfaces. With the less active metals the readings agreed to about 0.1° which closer examination showed was instrumental variation. Table I gives the original observations on four silver surfaces evaporated from a graphite crucible. The time required for this set of measurements as well as recoating both surfaces was less than fifteen minutes. The deviations of these observations from their mean indicates a probable error of about 0.1° in their mean. It was essential that the mirror *m* reflected the light back along its original path so that the two reflections from the metal mirror took place at the same angle of incidence. This was accomplished by inserting a pinhole in front of the window *W* and adjusting *m* until the light reflected from it returned through the pinhole. Before the chamber was evacuated pieces of plate glass were inserted for both mirrors and adjusted until all the reflected images of cross hairs or a pinhole coincided. These two adjustments made certain that the two mirrors were parallel to the axis of rotation and the light retraced its original path through the Nicol. A

TABLE I. *Extinction angles for silver at 5461A.*

	INCIDENCE		AZIMUTH	
	156.1		281.0	8.8
		125.2	100.7	8.3
	336.0		280.8	188.8
		305.4	281.0	188.4
Surfaces recoated				
	155.8		100.9	8.8
		125.2	281.3	188.7
	336.2		281.1	188.4
		305.2	100.9	8.6
Averages	156.03	305.25	100.96	188.60
Differences	$2\phi=149.22$		$2\psi=87.64$	
	$\phi=74.6°$		$\psi=43.8°$	
Minor's[6] values for 5500A	$\phi=74.35°$		$\psi=43.36°$	

[6] Minor, Ann. d. Physik **10**, 581 (1903).

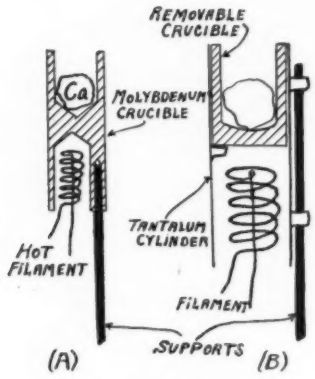

FIG. 2. Evaporating units.

Nicol of rhomboidal section was used to eliminate surface reflections and the lateral displacement due to the rotation of the Nicol produced a desirable scanning of the reflecting surface. The motion of the image of a distant lamp filament reflected through this optical system as both the Nicol and the mirror system were rotated showed that no deviation of as much as 0.1° was introduced by optical or mechanical defects.

With this method shown satisfactory and convenient for evaporated silver surfaces measurements were made on ten other elements most of which have not been studied by any other observer. Since it was convenient to take readings on several coatings on each side of the mirror, an evaporating unit was needed which could carry enough material for at least six opaque coatings. The use of small helical or conical tungsten filaments was very inefficient and cumbersome. Small molybdenum crucibles similar to those described by Rudberg[7] and heated by electron bombardment and radiation were found to be more satisfactory. Two types of these are shown in Fig. 2. The 17-mil tungsten filaments were wound on a mandril made of a machine screw with a small hole drilled along its axis. The type (*A*) were made of molybdenum rod, 6 mm in diameter and 15 mm long. Stiff tungsten supports carried the filaments and crucible. These were held together by a Pyrex bead several centimeters away from the crucible. All of the alkaline earth metals except beryllium evaporated readily without the need of electron bombardment. Other elements which alloyed with molybdenum and not with graphite, tantalum or alundum were evaporated from furnaces of the type (*B*). Crucibles containing any element could be placed in the cylinder of tantalum or molybdenum without disturbing the filament. Cylindrical radiation shields of larger diameter were desirable for the highest temperatures. Aluminum, cerium, and lanthanum alloyed with all available crucibles and had to be evaporated directly from tungsten filaments. Both the first and last bit of metal distilled from either a crucible or filament produced an undesirable surface. Calcium and the other very oxidizable elements had to be evaporated very rapidly in the highest possible vacuum to obtain consistent results. Optical constants measured during the process of rapid evaporation, immediately after, and an hour later showed no differences for these metals. The slightest trace of milkiness or darkness on a surface led to great differences. It was impossible to obtain reliable measurements on indium, thallium and the alkali metals at room temperature for this reason.

The angles of principal incidence and azimuth obtained by this method for a number of elements together with the optical constants computed from them are shown in Table II. None of these elements gave readings as reproducible as those for silver in Table I. In no instance are the angles certain to less than 0.1° but it is hoped that all are within 0.5°. The optical constants were calculated from these angles by means of Kettler's equations which are presented in a very understandable manner by König.[8] None of the generally accepted approximations have been used. The equations used to calculate the refractive index n, the extinction coefficient k, and the reflecting power at normal incidence R are

$$n^2k=\sin^2\phi\tan^2\phi\sin 2\psi\cos 2\psi, \tag{1}$$

$$n^2=\tfrac{1}{2}\tan^2\phi(1-2\sin^2\phi\sin^2 2\psi + (1-\sin^2 2\phi\sin^2 2\psi)^{\frac{1}{2}}), \tag{2}$$

and

$$R=\frac{(n-1)^2+(nk)^2}{(n+1)^2+(nk)^2}. \tag{3}$$

[7] Rudberg, Phys. Rev. **46**, 763 (1934).

[8] König, *Handbuch der Physik* (1928), Vol. 20, p. 245.

TABLE II. *The optical constants of metals in vacuum.*

	ϕ	2ψ	n	k	nk	R	$n^2(1-k^2)$	$n^2k\nu/N$
Beryllium								$\times 10^{-8}$
5780	74.5	42.2	2.64	0.86	2.27	43	1.9	2.52
5461	74.5	42.4	2.66	.89	2.36	44	1.5	2.79
4916	74.4	43.1	2.64	.85	2.25	42	1.9	2.92
4358	74.1	43.4	2.56	.87	2.23	42	1.5	3.15
4046	73.8	44.1	2.48	.89	2.20	42	1.3	3.26
Magnesium								
5780	75.9	83.1	0.48	7.74	3.71	88	−13.5	2.13
5461	75.3	81.6	.57	6.14	3.47	85	−11.7	2.49
4916	73.0	80.5	.53	5.41	2.92	81	− 8.2	2.23
4358	71.6	80.1	.52	5.09	2.65	76	− 6.8	2.34
4046	71.2	79.8	.52	4.94	2.05	68	− 3.9	2.48
Calcium								
5780	69.4	83.7	.29	7.94	2.31	83	− 5.3	1.73
5461	68.5	84.0	.27	8.08	2.16	83	− 4.6	1.38
4916	66.9	83.1	.29	6.64	1.92	78	− 3.6	1.48
4358	64.3	82.0	.29	5.60	1.64	73	− 2.6	1.44
4046	63.6	80.6	.34	4.53	1.56	68	− 2.3	1.70
Strontium								
5780	68.6	76.3	.61	3.50	2.13	66	− 4.2	3.74
5461	67.7	75.2	.63	3.15	1.99	60	− 3.6	3.83
4916	64.5	74.4	.58	2.78	1.61	55	− 2.4	3.95
4358	63.5	73.9	.57	2.61	1.50	51	− 1.9	3.27
4046	61.1	73.0	.55	2.32	1.28	46	− 1.3	3.95
Barium								
5780	64.5	65.7	.88	1.73	1.52	40	− 1.5	4.46
5461	64.4	65.4	.89	1.71	1.51	40	− 1.5	4.72
4916	61.9	63.7	.86	1.48	1.26	32	− 0.9	4.24
4358	59.7	64.2	.78	1.42	1.10	28	− .6	3.77
4046	59.1	63.4	.82	1.23	1.07	26	− .5	3.91
Aluminum								
5780	80.3	81.2	.89	6.29	5.62	90	−31	4.30
5461	80.1	81.9	.81	6.77	5.47	90	−30	4.01
4916	78.8	82.3	.68	7.08	4.80	90	−23	3.29
4358	77.6	83.0	.55	7.77	4.23	90	−19	2.71
4046	76.6	83.8	.45	8.85	3.98	90	−16	2.19
Germanium								
5780	75.5	21.6	3.42	.39	1.35	36	9.7	5.34
5461	75.2	23.4	3.47	.40	1.40	37	10.1	6.09
4916	74.1	26.0	3.16	.45	1.42	34	7.9	6.11
4358	73.8	31.8	2.93	.57	1.67	34	5.8	7.50
4046	73.3	34.4	2.85	.58	1.67	35	5.4	7.86
Lanthanum								
5780	75.6	61.1	1.74	1.99	3.47	64	− 9.0	11.9
5461	75.7	60.8	1.79	1.91	3.43	63	− 8.6	12.5
4358	72.0	64.1	1.35	1.83	2.49	54	− 4.4	8.3
4046	71.5	64.4	1.34	1.74	2.33	51	− 3.7	8.4
Cerium								
5780	73.6	55.8	1.91	1.35	2.58	50	− 3.0	8.6
5461	72.4	56.7	1.74	1.38	2.39	47	− 2.7	7.6
4358	69.4	58.3	1.41	1.40	1.97	42	− 1.9	6.4
Manganese								
5780	76.4	51.3	2.59	1.18	3.04	53	− 2.5	5.2
5461	76.2	52.9	2.46	1.25	3.07	54	− 3.3	5.2
4358	74.1	53.8	2.08	1.26	2.62	50	− 2.5	4.7

The aluminum as Gaviola and Strong[9] have shown contains a trace of tungsten from the filament and the constants refer to such an alloy of homogeneous composition. The results are quite different from those of Drude[1] and in close agreement with the remarkably high reflectivity of freshly evaporated surfaces of this element. The reflecting powers of magnesium and calcium are considerably higher than the other metals of the alkaline earth group. The trend of their optical constants indicate higher reflecting powers for longer wave-lengths. Preliminary observations on silicon and germanium films with a rocksalt spectrometer show high infrared transmission for films of these two elements which are quite opaque to visible light. Germanium has the advantage of being readily evaporated from either a graphite crucible or a tungsten filament.

Also listed in Table II and plotted in Fig. 3 are the dielectric constants and atomic rates of absorption which are the two optical constants most directly related to the distribution of energy states in the metal. The dielectric constant of an absorbing medium indicates the strength and nearness of absorption bands by its magnitude and algebraic sign. The expression relating the frequency of the absorption band ν_0, the frequency of the radiation ν, the oscillator strength df electrons per atom, and the dielectric constant ϵ is

$$\epsilon = 1 - \frac{Ne}{m}\int \frac{df}{\nu^2 - \nu_0^2}, \tag{4}$$

where

$$\epsilon = n^2(1-k^2), \tag{5}$$

$$f = \frac{4m}{Ne^2}\int n^2 k\nu \cdot d\nu, \tag{6}$$

and

$$n^2 k\nu = \sigma. \tag{7}$$

The other constants appearing in these expressions are the mass of the electron m, its charge e, and N the number of atoms per cubic centimeter of the metal. The terms dielectric constant and conductivity have been applied to the expressions (5) and (7) as these are the coefficients of the displacement and conduction currents in Maxwell's equations for the propagation of an electromagnetic wave in an absorbing medium. If one computes[10] the rate of absorption of energy per second per cubic centimeter from a plane wave of unit amplitude of the electric field within the metal the result is identical with (7). This is most readily obtained from differentiating the Poynting expression for the plane wave within

[9] Gaviola and Strong, Phys. Rev. **48**, 136 (1935).

[10] Schaefer, *Maxwell's Théorie* (1908), p. 162.

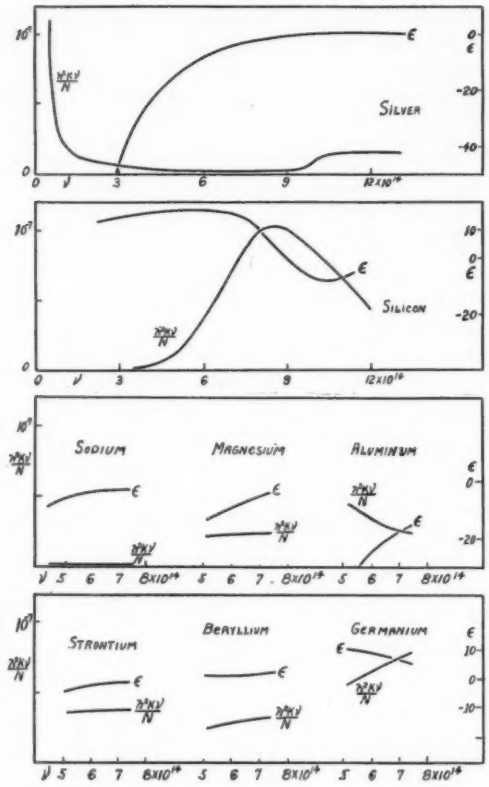

FIG. 3. Optical constants of metals plotted against frequency.

the metal with respect to the direction of propagation. If this result is divided by N a final expression is found which is the energy absorbed per second by an atom in an electric field of unit electric intensity and of the frequency ν. This last quantity is the one most directly related to the energy states of the metal and is called for short the atomic rate of absorption.

Unfortunately the distribution of filled energy states, unoccupied states, and transition probabilities between these combine to determine atomic rate of absorption of a metal for radiation of a particular frequency. To separate partially the parts played by these three factors in determining the optical constants the dielectric constant and atomic rate of absorption of silver and silicon have been plotted in Fig. 3. These two elements have been chosen since they represent the very good electrical conductors with one metallic electron and the almost nonconducting class of elements. The optical constants of the two substances have been measured over a very extensive region of the spectrum by various observers.[6, 11] The curves for silver are remarkably simple; showing two regions of high atomic rate of absorption one in the ultraviolet and the other near zero frequency, with the dielectric constant behaving in accord with expression (4) and two such absorption bands. Copper and gold differ from silver only in the strengths of the regions of absorption and the location of the high frequency absorption. Zener[12] has been able to calculate the ultraviolet transparency of the alkali metals assuming only one absorption band which is at zero frequency. This is possible because the ultraviolet absorption is very weak and indicates that alloys of two alkali metals should have their transmission at some wave-length between those of the separate components. Sommerfeld and Bethe[13] have computed the strength of the oscillator at zero frequency necessary to give the dielectric constant shown by silver in the infrared and found 0.8 electron per atom. Applying the f sum rule which states that the value of f computed from (6) over the entire region where the metallic electrons shown absorption should equal the number of these electrons, the ultraviolet absorption of silver should not exceed 0.2 electron. The ultraviolet absorption in Fig. 3 amounts to almost this much with probably a little more occurring at still shorter wave-lengths.

On the other hand silicon shows absorption throughout the visible and ultraviolet amounting to about 1.6 electrons while a total of 2 should be found. The positive value of the dielectric constant for the infrared indicates a very small oscillator strength of zero frequency so that the residual absorption must lie in the extreme ultraviolet. The theory presented by Sommerfeld and Bethe[13] describes satisfactorily absorption near zero frequency as due to that portion of the

[11] Ingersoll, Astrophys. J. **32**, 286 (1910); Fosterling and Freedericksz, Ann. d. Physik **40**, 201 (1913); Pfesdorf, Ann. d. Physik **81**, 906 (1926).

[12] Zener, Nature **132**, 968 (1933); Mott and Zener, Proc. Camb. Phil. Soc. **2**, 249 (1934).

[13] Sommerfeld and Bethe, *Handbuch der Physik* (1933), Vol. 24, part 2.

electrons which are free, the ultraviolet absorption by quantum transitions permitted by the selection rules, and the small visible absorption on account of irregularities in the metal crystal and collisions with the thermal vibrations of the atoms but this description is limited to metals with one metallic electron. A more exact calculation for certain of these metals has been made by Wilson[14] and applications of this type of theory to supraconductivity has been made by Kronig[15] although no metals with only one electron per atom have been found to be supraconductors. The dielectric constant and atomic rate of absorption of sodium for the visible region from Duncan's data[3] and of several other metals from the present paper are shown in Fig. 3. Those metals with two, three or four electrons per atom show much higher atomic rates of absorption than the one electron elements indicating much greater quantum absorption. The dielectric constants of magnesium and aluminum, the only elements of this study known to be supraconducting, indicate large oscillator strengths of zero frequency while germanium and possibly beryllium have a very small portion of their electrons free to respond to electric fields of near zero frequency. This is in general qualitative agreement with their electrical conductivity.

The general theory of metals with two, three, or four electrons per atom predicts complicated optical and electrical properties and it is among these that all the supraconducting elements are found. Unlike silver, which has the lowest band of energy levels only partially filled, these metals with more than one valence electron have the lowest band completely filled and in some instances the second band is partially filled. The soft x-rays emitted by beryllium, magnesium, aluminum and silicon[16] show the presence of a sparcity of occupied levels of certain energies. It would be very desirable to extend these measurements into the ultraviolet and infrared regions for the five alkaline earth metals which differ so greatly in the visible. The infrared is probably more interesting especially for those elements which show supraconductivity. Experimentally this would be possible with a photoelectric cell or thermocouple with an amplifier but would be very laborious compared to observations made visually.

[14] Wilson, Proc. Roy. Soc. **A151**, 274 (1935).
[15] Kronig, Proc. Roy. Soc. **A152**, 16 (1935).

[16] O'Bryan and Skinner, Phys. Rev. **45**, 370 (1934).

Approximations to the Cartesian Oval

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IN attempting to verify the analysis in an interesting paper[1] by Ludwik Silberstein the writer found that Eqs. (1), (7), (8), (9), (10), (11), and (12) are incorrect. The middle member of (10a) and the unnumbered relation $d^2x/dy^2 = 2x' + 4y^2 + x''$ are also wrong. It was noticed that by avoiding radicals and derivatives the desired algebraic expressions can be obtained, without disproportionate labor, with greater generality and in forms which are appreciably more compact and practical.

The context shows that Eq. (1) was intended to be

$$\mu[(l_0+x)^2+y^2]^{\frac{1}{2}}+\mu'[(l-x)^2+y^2]^{\frac{1}{2}}=\mu' l+\mu l_0, \quad (1_c)$$

since l_0 was tacitly assumed to be positive when measured from the point P_0 to O. [Equations in the present commentary will be distinguished from those in Silberstein's paper by affixing the subscript c to the number of the equation.]

Let the equation of the meridian section of the Cartesian surface be written

$$a_0x^4+2a_0x^2y^2+a_0y^4+a_1x^3+a_1xy^2 +a_2x^2+a_3y^2+a_4x=0. \quad (2_c)$$

By identifying the coefficients in this equation with the corresponding ones in the equation obtained by rationalizing (1_c) it is found that

$$\begin{aligned} a_0&=(\mu'^2-\mu^2)^2,\\ a_1&=-4(\mu'^2-\mu^2)(\mu'^2 l+\mu^2 l_0),\\ a_2&=4[(\mu'^2 l+\mu^2 l_0)^2-\mu'\mu(\mu' l+\mu l_0)(\mu' l_0+\mu l)], \quad (3_c)\\ a_3&=-4\mu'\mu(\mu' l+\mu l_0)(\mu' l_0+\mu l),\\ a_4&=8\mu'\mu l l_0(\mu'-\mu)(\mu' l+\mu l_0). \end{aligned}$$

Hence $$a_1^2=4a_0(a_2-a_3). \quad (4_c)$$

Let $$x=\beta_2y^2+\beta_4y^4+\beta_6y^6+\beta_8y^8+\cdots. \quad (5_c)$$

Substitute this series in Eq. (2_c) and equate to zero the resulting coefficients of y^2, y^4, y^6, y^8, etc., to find

$$a_3+a_4\beta_2=0, \quad (6_c)$$

$$a_0+a_1\beta_2+a_2\beta_2^2+a_4\beta_4=0, \quad (7_c)$$

$$2a_0\beta_2^2+a_1\beta_2^3+a_1\beta_4+2a_2\beta_2\beta_4+a_4\beta_6=0, \quad (8_c)$$

$$a_0\beta_2^4+4a_0\beta_2\beta_4+3a_1\beta_2^2\beta_4+a_1\beta_6 +a_2(\beta_4^2+2\beta_2\beta_6)+a_4\beta_8=0. \quad (9_c)$$

By Eq. (6_c) $\beta_2=-a_3/a_4$; hence, by substituting from (3_c),

$$\beta_2=(\mu' l_0+\mu l)/2l_0 l(\mu'-\mu). \quad (10_c)$$

With reference to Silberstein's symbols $\rho=2x_0'=2\beta_2$, accordingly comparison of formulas (8) and (10_c) shows that the denominator of (8) should have been $\mu'-\mu$ instead of $\mu+\mu'$. Unfortunately formula (10) cannot be corrected by the simple expedient of replacing $\mu+\mu'$ by $\mu'-\mu$ throughout.

In the notation used in the present analysis Eq. (6) becomes

$$\Delta x=(\beta_4-\beta_2^3)y^4+(\beta_6-2\beta_2^5)y^6 +(\beta_8-5\beta_2^7)y^8+\cdots, \quad (6')$$

in which the coefficients must now be determined as explicit functions of μ', μ, l_0, and l.

Elimination of β_2 between Eqs. (6_c) and (7_c) leads to

$$\beta_4=-(a_0a_4^2-a_1a_3a_4+a_2a_3^2)/a_4^3. \quad (11_c)$$

Now

$$\beta_4-\beta_2^3=-[a_0a_4^2-a_1a_3a_4+(a_2-a_3)a_3^2]/a_4^3$$

and, by (4_c),

$$a_2-a_3=a_1^2/4a_0;$$

hence

$$\beta_4-\beta_2^3=-(2a_0a_4-a_1a_3)^2/4a_0a_4^3. \quad (12_c)$$

By (3_c)

$$2a_0a_4-a_1a_3 =-16\mu'^2\mu^2(\mu'^2-\mu^2)(l+l_0)(\mu' l+\mu l_0)^2, \quad (13_c)$$

so that, up to the sixth order in y,

$$\begin{aligned} \Delta x&=(\beta_4-\beta_2^3)y^4\\ &=-\mu'\mu(l+l_0)^2(\mu' l+\mu l_0)y^4/8l^3l_0^3(\mu'-\mu)^3. \quad (14_c) \end{aligned}$$

Formula (14_c) is the correct expression to replace the original (10). The new equation has

[1] Silberstein, J. O. S. A. **25**, 340 (1935).

the very appreciable advantage of possessing a form suitable for logarithmic computation. It also involves only the given quantities μ', μ, l_0, and l (but not ρ). When $\mu'-\mu$ and $\mu' l+\mu l_0$ have the same sign Eq. (14_c) proves that Δx is negative, that is to say, the osculating sphere at P_0 lies on the concave side of the Cartesian surface. This fact constitutes a generalization of the special case of the ellipsoid referred to by Silberstein just below Eq. (10a).

By using Eqs. (4_c), (6_c), and (7_c) to eliminate a_2, β_2 and β_4, respectively, from relation (8_c) it results that

$$\beta_6-2\beta_2{}^5=-(2a_0a_4-a_1a_3)^2 \times(8a_0a_3{}^2-2a_0a_1a_4+a_1{}^2a_3)/8a_0{}^2a_4{}^5 \quad (15_c)$$

or

$$\Delta x=(\beta_4-\beta_2{}^3)y^4-\mu'\mu(l+l_0)^2 \times(\mu' l+\mu l_0)[2(\mu' l_0+\mu l)^2-(l+l_0) \times(\mu'{}^2 l+\mu^2 l_0)]y^6/16l^5l_0{}^5(\mu'-\mu)^5. \quad (16_c)$$

Formula (16_c) supersedes (11) of the earlier paper. It is interesting to note that under special conditions the trinomial factor of expression (15_c) can vanish. If μ' and μ are given, the ratio of l to l_0 is determined. More precisely

$$l/l_0=\{\pm(\mu'-\mu)[\mu'{}^2+\mu^2+5(\mu'+\mu)^2]^{\frac{1}{2}} -\mu'{}^2-\mu^2+4\mu'\mu\}/2(\mu'{}^2-2\mu^2). \quad (17_c)$$

where the discriminant of the quadratic has been so written as to show that the roots would remain real even if μ' or μ were negative. For $\mu'=3/2$ and $\mu=1$ the positive root of (17_c) is

$$l/l_0=((177)^{\frac{1}{2}}+11)/2=12.15207.$$

Incidentally there is a misprint in the unnumbered equation below (11a), the denominator should be $16n^4$ instead of $16n^2$.

By applying the preceding method to the determination of the coefficient of y^5 in Eq. (6′) it can be shown that

$$\beta_3-5\beta_2{}^7=-5(2a_0a_4-a_1a_3)^2 \times\{(8a_0a_3{}^2-2a_0a_1a_4+a_1{}^2a_3)^2 +16a_0{}^2a_3(a_0a_4{}^2-a_2a_3{}^2)\}/64a_0{}^3a_4{}^7. \quad (18_c)$$

After extended analysis beginning with expression (18_c) the explicit function of μ', μ, l, and l_0 in the following formula will be obtained:

$$\Delta x=(\beta_4-\beta_2{}^3)y^4+(\beta_6-2\beta_2{}^5)y^6 -5\mu'\mu(l+l_0)^2(\mu' l+\mu l_0)\{2(\mu' l_0+\mu l)^4 +[(l+l_0)(\mu'{}^2 l+\mu^2 l_0)-(\mu' l_0+\mu l)^2]^2 -\mu'\mu(l+l_0)^2[ll_0(\mu'-\mu)^2 +\mu'\mu(l+l_0)^2]\}y^8/128l^7l_0{}^7(\mu'-\mu)^7. \quad (19_c)$$

In applying the formulas given in the present paper to concrete cases the computer may find it easier first to calculate the a's from the set of expressions (3_c) and then to evaluate relations (12_c), (15_c), and (18_c) than to substitute numbers for μ', μ, l and l_0 in the final formulas (14_c), (16_c), and (19_c).

Since the first three or four terms of an infinite series do not in general show whether the series is convergent, or asymptotic, or divergent it seemed to the writer to be desirable to test this question in the two special cases in which an exact solution can be easily obtained and interpreted. These cases arise for $1/l_0=0$ and $1/l=0$.

When $1/l_0=0$ Eq. (2_c) reduces to

$$(\mu'{}^2-\mu^2)x^2+\mu'{}^2y^2-2\mu' l(\mu'-\mu)x=0, \quad (20_c)$$

which, for $\mu'>\mu$, clearly represents an ellipse of semi-major axis $\mu' l/(\mu'+\mu)$, semi-minor axis $l(\mu'-\mu)^{\frac{1}{2}}/(\mu'+\mu)^{\frac{1}{2}}$, and center $x=\mu' l/(\mu'+\mu)$, $y=0$. The equation of the osculating circle at the point P_0 is

$$\mu' x^2+\mu' y^2-2(\mu'-\mu)lx=0, \quad (21_c)$$

the radius being $R=l(\mu'-\mu)/\mu'$.

Let x_e and x_c denote the smaller roots obtained, respectively, by solving the quadratic Eqs. (20_c) and (21_c) for x in terms of the same value of y, and choose $y=R/m=l(\mu'-\mu)/m\mu'$. The following formula for the difference x_e-x_c is exact:

$$(x_e-x_c)/R=-\{n^2(m^2n^2-n^2+1)^{\frac{1}{2}} -n(n^2-1)(m^2-1)^{\frac{1}{2}}-mn\}/mn(n^2-1), \quad (22_c)$$

where $n\equiv\mu'/\mu$. The corresponding approximation formula which includes y^6 but not y^8 may be written

$$\Delta x/R\doteq-(2m^2n^2+2n^2-1)/16m^6n^4. \quad (23_c)$$

For $m=10$ and $n=3/2$ formula (22_c) gives 1/178,600.4 for $(x_c-x_e)/R$. The negative of the approximation expression (23_c) equals 1/178,610.8 hence the omitted terms of the series would contribute 0.0058 percent.

For $m=5$ and $n=3/2$ the numbers obtained from the exact and approximate difference formulas are respectively 1/10,900.40 and 1/10,910.56 the discrepancy between which is 0.093 percent.

Another pertinent test of the formulas of approximation is afforded by letting $1/l$ approach zero. In this case the plane wave fronts will lie in the medium of index μ'. Eq. (2_c) now reduces to

$$(\mu'^2-\mu^2)x^2-\mu^2y^2+2\mu l_0(\mu'-\mu)=0, \quad (24_c)$$

and the meridian curve is, for $\mu'>\mu$, an hyperbola of semi-major axis $\mu l_0/(\mu'+\mu)$, semi-minor axis $l_0(\mu'-\mu)^{\frac{1}{2}}/(\mu'+\mu)^{\frac{1}{2}}$, and center $x=-\mu l_0/(\mu'+\mu)$, $y=0$. The equation of the osculating circle at the vertex is

$$\mu x^2+\mu y^2-2l_0(\mu'-\mu)x=0. \quad (25_c)$$

Proceeding as in the preceding case of the ellipse it is found that, for the common ordinate $y=l_0(\mu'-\mu)/m\mu$,

$$(x_h-x_c)/R=-\{mn^2-(m^2+n^2-1)^{\frac{1}{2}} -(n^2-1)(m^2-1)^{\frac{1}{2}}\}/m(n^2-1). \quad (26_c)$$

Through the term in y^6 the corresponding approximation formula is

$$\Delta x/R \doteqdot -n^2(2m^2-n^2+2)/16m^6. \quad (27_c)$$

For $m=10$ and $n=3/2$ formula (26_c) gives 1/35,598.60 for $(x_c-x_h)/R$. The negative of the expression (27_c) equals 1/35,600.06 hence the omitted terms of the series would contribute 0.0041 percent.

For $m=5$ and $n=3/2$ the numbers derived from the exact and approximate difference formulas are respectively 1/2231.936 and 1/2233.389 the discrepancy between which is 0.065 percent.

In conformity with Silberstein's notation

$$C_8/C_6=(5/8)(y/R)^2(3n^4-3n^2+1)/[n^2(2n^2-1)] \quad (28_c)$$

for the limiting case $1/l_0=0$. When $n=3/2$ and $y/R=1/10$ Eq. (28_c) gives $C_8/C_6=151/20{,}160=1/133.5$ while $C_6/C_4=7/900=1/128.6$.

Similarly for $1/l=0$

$$C_8/C_6=(5/8)(y/R)^2(n^4-3n^2+3)/(2-n^2). \quad (29_c)$$

This relation becomes $C_8/C_6=-21/640=-1/30.48$ when $n=3/2$ and $y/R=1/10$. Simultaneously $C_6/C_4=-1/800$. The last two ratios illustrate the fact, referred to earlier, that inferences suggested by the first few terms of an infinite series may be fallacious. Thus for $n=\sqrt{2}=1.4142$ (a perfectly normal value) C_8/C_6 becomes infinite whereas $-1/35{,}598.6$ is the limit of the sum of the series. In general the fulfillment of condition (17_c) is accompanied by an infinite value of the ratio C_8/C_6.

A special case which is instructive and affords an additional check on the analysis arises when $\mu' l+\mu l_0=0$ (l negative, image virtual). Then $a_3=a_4=0$ [see set (3_c)] and the general Eq. (2_c) reduces to

$$\{2a_0(x^2+y^2)+a_1x\}^2=0, \quad (30_c)$$

which shows that the two ovaliform surfaces have coalesced and made infinitely perfect contact with the osculating sphere through the point P_0. It is customary in treatises[2] on geometrical optics to consider the sphere (or circle) as given and to use Snell's law of refraction in establishing the existence and investigating the properties of the "aplanatic" pair of points. It can be proved from the present analysis that the relation $\mu \sin i=\mu' \sin r$ must hold. It should now be clear why the factors $(l+l_0)^2$ and $\mu' l+\mu l_0$ do and must occur in (14_c), (16_c), (19_c) and in the coefficients of all powers of y^2 in series (6').

In conclusion the writer desires to thank Mr. Howard L. Schultz, Francis E. Loomis, Fellow in Yale University, for having independently verified all of the analysis involved and the numerical results given in this commentary.

[2] See, for example, James P. C. Southall's *The Principles and Methods of Geometrical Optics*, second edition, Arts. 207, 211. Note 3, and 267.

Corrigenda to "A Simplified Computation of Cartesian Lens Surfaces"*

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IN the right-hand member of Eq. (1) read $(x+l_0)^2$ instead of $(x-l_0)^2$, the sign of l_0 in the original paper having been changed by inadvertence.

This leaves Eqs. (2), (3), (4), (5), (6) unaffected, but the first term in Eq. (7) has to be replaced by

$$\mu[1+2(x+l_0)x']/[(x+l_0)^2+u]^{\frac{1}{2}}$$

and the denominator $(\mu+\mu')$ in Eq. (8) has to be replaced by $(\mu'-\mu)$. Further, Eq. (9) should read

$$x_0''=\frac{1}{\mu-\mu'}\left[\left(\frac{\mu}{l_0^2}-\frac{\mu'}{l^2}\right)\frac{\rho}{2}-\frac{1}{4}\left(\frac{\mu}{l_0^3}+\frac{\mu'}{l^3}\right)\right]$$

and, in Eq. (10), $(\mu/l_0^3+\mu'/l^3)$ has to be changed into $(\mu/l_0^2-\mu'/l^2)$. The modification of (10a) can be left to the reader, since (10a) is only a subcase of (10), *viz.*, for $l_0=\infty$.

Independently of these simple consequences of the inversion of the sign of l_0 there is a typographical error in the unnumbered equation just below (11a): In the right-hand member of that equation read $16n^4$ instead of $16n^2$.

* J. O. S. A. **25**, 340 (1935).

Necrology

C. W. Robbins

MR. C. W. Robbins was born at Bradyville, Iowa, August 24, 1875. He received his grammar school education at Humboldt, Nebraska, at which place he also finished a high school course in 1891.

After having some experience in inspection work and in electrical engineering, he was employed by the Western Electric Company in October, 1905, as an inspector. It was soon apparent that his abilities were far in excess of those required for that type of work. In view of his well-founded knowledge in electrical engineering, he was promoted to work involving the development of inspection methods. Here he again demonstrated that his talents were in excess of those required for an individual worker and due to his supervisory and managerial ability, he rose step by step until in March, 1926, he was made Assistant Superintendent of Inspection Engineering Development work.

Mr. Robbins continued in the capacity of Assistant Superintendent on various types of engineering work until he retired from active service on August 1, 1933.

Late in 1934 he was suddenly stricken by an affliction, the nature of which was never fully determined. He did not recover from this malady and died on December 13, 1934. He was survived by his widow.

Mr. Robbins was elected to membership in the Optical Society in 1929. He was deeply interested in sound and light but most particularly in the latter. He was well versed in optical matters and in consequence he possessed a considerable amount of optical equipment. He was responsible for the introduction of a considerable amount of optical equipment for use in inspection work associated with the manufacture of telephone equipment.

W. J. TIETZ